A curve has equation

$$y = 3x^2 + \frac{24}{x} + 2 \qquad x > 0$$

(a) Find, in simplest form, $\frac{\mathrm{d}y}{\mathrm{d}x}$

(3)

(b) Hence find the exact range of values of *x* for which the curve is increasing.

(2)

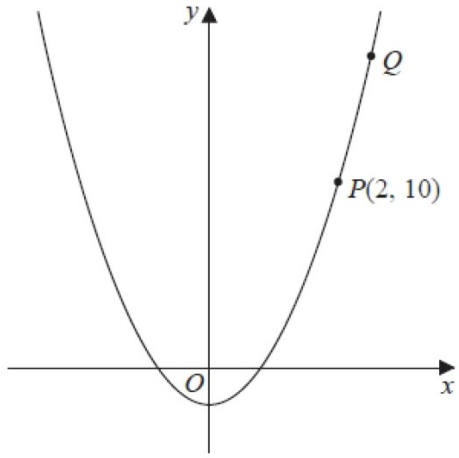


Figure 1

Figure 1 shows part of the curve with equation $y = 3x^2 - 2$

The point $P(2, 10)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at P.

(2)

The point Q with x coordinate $2 + h$ also lies on the curve.

(b) Find the gradient of the line PQ, giving your answer in terms of h in simplest form.

(3)

(c) Explain briefly the relationship between part (b) and the answer to part (a).

(1)

The curve C has equation $y = f(x)$ where

$$f(x) = ax^3 + 15x^2 - 39x + b$$

and a and b are constants.

Given

- the point (2, 10) lies on C
- the gradient of the curve at (2, 10) is -3

(a) (i) show that the value of a is -2

(ii) find the value of b.

(4)

(b) Hence show that C has no stationary points.

(3)

(c) Write $f(x)$ in the form $(x - 4)Q(x)$ where $Q(x)$ is a quadratic expression to be found.

(2)

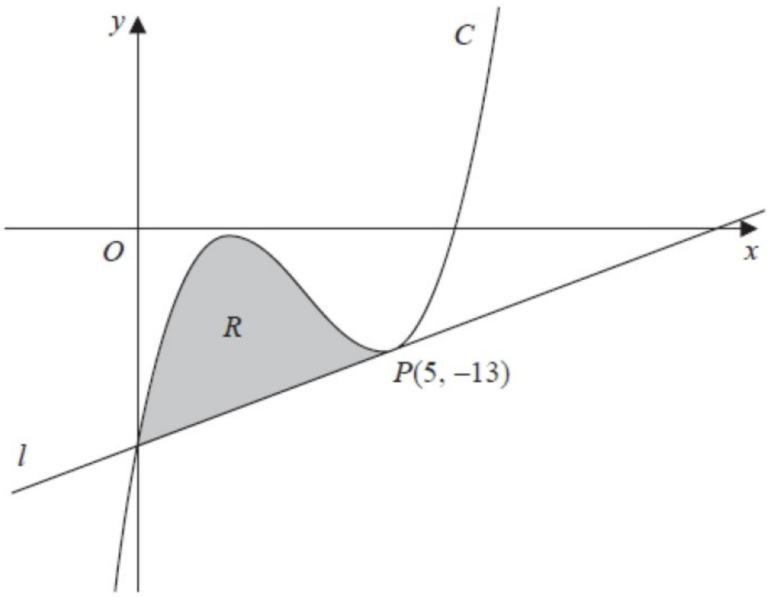


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + 27x - 23$$

The point $P(5, -13)$ lies on C

The line l is the tangent to C at P

(a) Use differentiation to find the equation of l, giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(4)

(b) Hence verify that l meets C again on the y-axis.

(1)

The finite region R, shown shaded in Figure 2, is bounded by the curve C and the line l.

(c) Use algebraic integration to find the exact area of R.

(4)

$$f(x) = \frac{(3 - 4\sqrt{x})^2}{\sqrt{x}}, \; x > 0$$

(a) Show that $f(x) = 9x^{-\frac{1}{2}} + Ax^{\frac{1}{2}} + B$, where A and B are constants to be found.

(3)

(b) Find $f'(x)$.

(3)

(c) Evaluate $f'(9)$.

(2)

(a) Write $\dfrac{2\sqrt{x}+3}{x}$ in the form $2x^p + 3x^q$ where p and q are constants.

(2)

Given that $y = 5x - 7 + \dfrac{2\sqrt{x}+3}{x}$, $x > 0$,

(b) find $\dfrac{dy}{dx}$, simplifying the coefficient of each term.

(4)

For the curve C with equation $y = \mathrm{f}(x)$,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^3 + 2x - 7.$$

(a) Find $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$.

(2)

(b) Show that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} \geq 2$ for all values of x.

(1)

Given that the point $P(2, 4)$ lies on C,

(c) find y in terms of x,

(5)

(d) find an equation for the normal to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

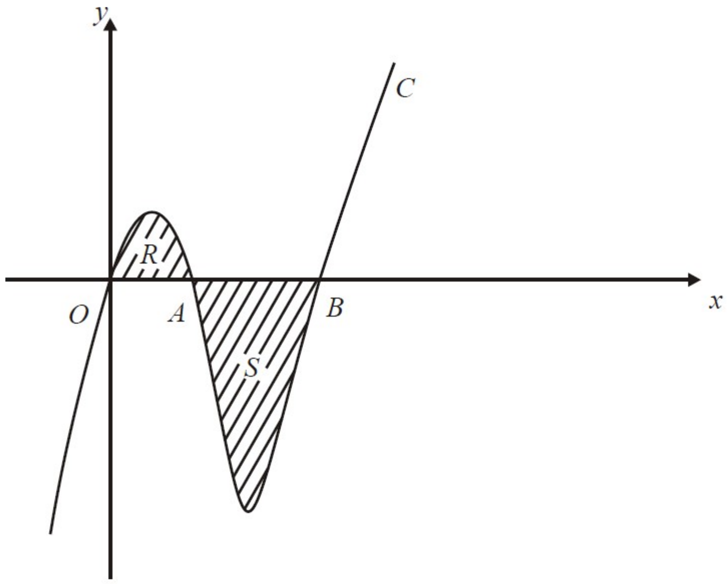
$$y = x^2 - k\sqrt{x}, \text{ where } k \text{ is a constant.}$$

(a) Find $\dfrac{dy}{dx}$.

(2)

(b) Given that y is decreasing at $x = 4$, find the set of possible values of k.

(2)



The diagram above shows part of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = x^3 - 6x^2 + 5x.$$

The curve crosses the x-axis at the origin O and at the points A and B.

(a) Factorise $\mathrm{f}(x)$ completely.

(3)

(b) Write down the x-coordinates of the points A and B.

(1)

(c) Find the gradient of C at A.

(3)

The region R is bounded by C and the line OA, and the region S is bounded by C and the line AB.

(d) Use integration to find the area of the combined regions R and S, shown shaded in the diagram above.

(7)

The function

$$f(x) = 2x^3 - 3x^2 - px + 6$$

has a turning point at $x = 2$

(i) Find the value of p. [3]

....

(ii) Find the range of values of x for which $f(x)$ is an increasing function. [4]

....

....

The curve C has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 2$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

Given that $y = 2x^2 - \frac{6}{x^3}$, $x \neq 0$,

(a) find $\frac{dy}{dx}$,

(2)

(b) evaluate $\int_1^3 y \, dx$.

(4)