A curve has equation

$$y = 3x^2 + \frac{24}{x} + 2 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(3)

(b) Hence find the exact range of values of $x$ for which the curve is increasing.

(2)

(a) $y = 3x^2 + 24x^{-1} + 2$

$\frac{dy}{dx} = 6x - 24x^{-2}$

$\frac{dy}{dx} = 6x - \frac{24}{x^2}$

(b) Curve is increasing when $\frac{dy}{dx} > 0$

$6x - \frac{24}{x^2} > 0$

$6x > \frac{24}{x^2}$

$x^3 > 4$

$x > \sqrt[3]{4}$

Answer $x > \sqrt[3]{4}$

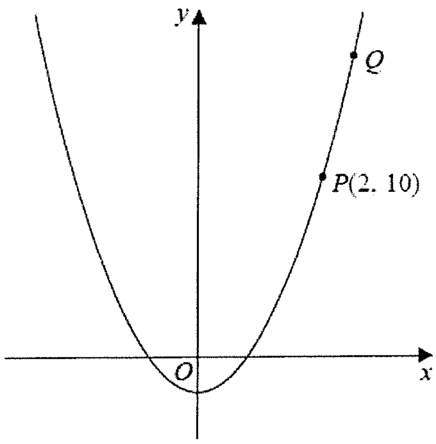



**Figure 1**

Figure 1 shows part of the curve with equation $y = 3x^2 - 2$

The point $P(2, 10)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at $P$.

(2)

The point $Q$ with $x$ coordinate $2 + h$ also lies on the curve.

(b) Find the gradient of the line $PQ$, giving your answer in terms of $h$ in simplest form.

(3)

(c) Explain briefly the relationship between part (b) and the answer to part (a).

(1)

$y = 3x^2 - 2$

$\frac{dy}{dx} = 6x$ $\quad$ $m = \text{gradient} = \frac{dy}{dx}$

At point $(2, 10)$

$m = 6 \times 2 = 12$ $\quad$ Gradient $= 12$

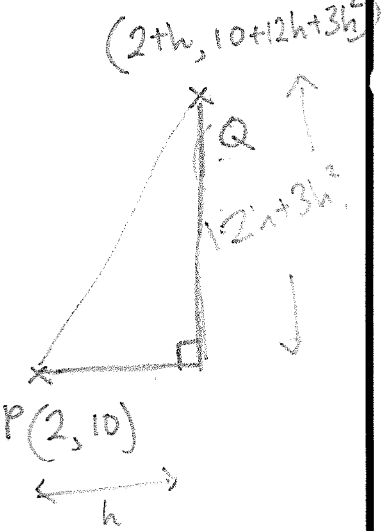
Point Q $= 3(2+h)^2 - 2$

$= 3(4 + 4h + h^2) - 2$

$= 12 + 12h + 3h^2 - 2$

$= 10 + 12h + 3h^2$

Point Q $(2+h, 10 + 12h + 3h^2)$




Gradient $= \frac{12h + 3h^2}{h}$

$= 12 + 3h$

As h gets smaller

Gradient $= 12 + 3h$

$= 12$

The curve $C$ has equation $y = f(x)$ where

$$f(x) = ax^3 + 15x^2 - 39x + b$$

and $a$ and $b$ are constants.

Given

- the point (2, 10) lies on $C$
- the gradient of the curve at (2, 10) is $-3$

(a) (i) show that the value of $a$ is $-2$

(ii) find the value of $b$.

(4)

(b) Hence show that $C$ has no stationary points.

(3)

(c) Write $f(x)$ in the form $(x - 4)Q(x)$ where $Q(x)$ is a quadratic expression to be found.

(2)

$f(x) = ax^3 + 15x^2 - 39x + b$

$f'(x) = 3ax^2 + 30x - 39$

Gradient at (2,10) is $-3$

$-3 = 3a(2)^2 + 30(2) - 39$

$-3 = 12a + 60 - 39$

$-24 = 12a$

$-2 = a$

$f(x) = -2x^3 + 15x^2 - 39x + b$

$f(2) = 10$

$10 = -2(2)^3 + 15(4) - 39(2) + b$

$10 = -16 + 60 - 78 + b$

$10 = -34 + b$

$44 = b$

Ans $b = 44$

$f(x) = -2x^3 + 15x^2 - 39x + 44$

$f'(x) = -6x^2 + 30x - 39$

Find turning points by $f'(x) = 0$

Long division

$-6x^2 + 30x - 39 = 0$

$-3(2x^2 - 10x + 13) = 0$

$a = 2$
$b = -10$
$c = 13$

$b^2 - 4ac$

$100 - 4(2)(13)$

$100 - 104$

No solutions.

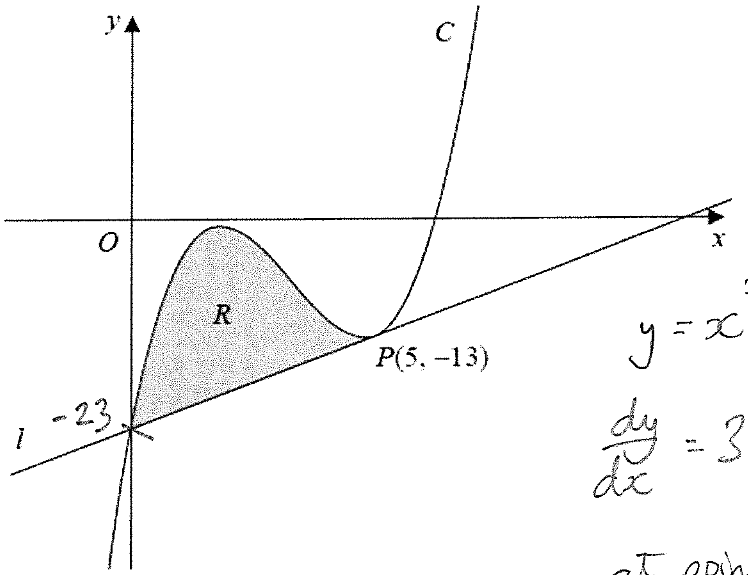



**Figure 2**

Figure 2 shows a sketch of part of the curve $C$ with equation

$$y = x^3 - 10x^2 + 27x - 23$$

The point $P(5, -13)$ lies on $C$

The line $l$ is the tangent to $C$ at $P$

(a) Use differentiation to find the equation of $l$, giving your answer in the form $y = mx + c$ where $m$ and $c$ are integers to be found.

(4)

(b) Hence verify that $l$ meets $C$ again on the $y$-axis.

(1)

The finite region $R$, shown shaded in Figure 2, is bounded by the curve $C$ and the line $l$.

(c) Use algebraic integration to find the exact area of $R$.

(4)

$y = x^3 - 10x^2 + 27x - 23$

$\frac{dy}{dx} = 3x^2 - 20x + 27$

at point $P(5,-13)$

$\frac{dy}{dx}$ = gradient

$= 3(5)^2 - 20(5) + 27$

$= 75 - 100 + 27$

$= 2$

(a) $y = 2x + c$

Put in $(5,-13)$

$-13 = 2(5) + c$

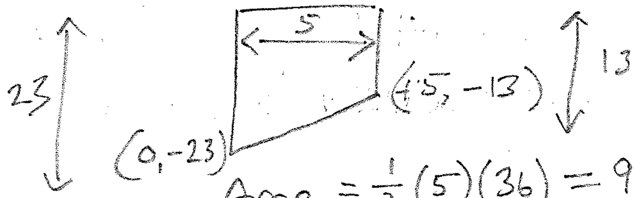
$-13 = 10 + c$

$-23 = c$

Ans $y = 2x - 23$

(b) I started trying to solve simultaneously. But it is much easier they both go through $(0,-23)$

(c) Find Area




Area $= \frac{1}{2}(5)(36) = 90$

Integrate

$$\int_0^5 (x^3 - 10x^2 + 27x - 23)\,dx = \frac{455}{12}$$

Area $R = 90 - \frac{455}{12}$

$$f(x) = \frac{(3-4\sqrt{x})^2}{\sqrt{x}}, \quad x > 0$$

(a) Show that $f(x) = 9x^{-\frac{1}{2}} + Ax^{\frac{1}{2}} + B$, where $A$ and $B$ are constants to be found.

**(3)**

(b) Find $f'(x)$.

**(3)**

(c) Evaluate $f'(9)$.

**(2)**

$$f(x) = \frac{(3-4\sqrt{x})(3-4\sqrt{x})}{\sqrt{x}} = \frac{9-24\sqrt{x}+16x}{\sqrt{x}}$$

$$f(x) = \frac{9}{\sqrt{x}} - 24 + 16\sqrt{x} \qquad A = +16 \qquad B = -24$$

$$9x^{-1/2} + 16x^{1/2} - 24$$

$$f'(x) = -\frac{9}{2}x^{-3/2} + 8x^{-1/2}$$

$$f'(x) = -\frac{9}{\sqrt{x^3}} + \frac{8}{\sqrt{x}}$$

$$f'(9) = -\frac{9}{\sqrt{9^3}} + \frac{8}{\sqrt{9}}$$

$$= -\frac{9}{27} + \frac{8}{3}$$

$$= -\frac{1}{3} + \frac{8}{3}$$

$$= \frac{7}{3}$$

$$= 2\frac{1}{3}$$

(a) Write $\frac{2\sqrt{x}+3}{x}$ in the form $2x^p + 3x^q$ where $p$ and $q$ are constants.

$p = -\frac{1}{2}$ $q = -1$ **(2)**

Given that $y = 5x - 7 + \frac{2\sqrt{x}+3}{x}$, $x > 0$,

(b) find $\frac{dy}{dx}$, simplifying the coefficient of each term.

**(4)**

(a) $\frac{2\sqrt{x}+3}{x} = \frac{2}{\sqrt{x}} + \frac{3}{x} = 2x^{-1/2} + 3x^{-1}$

$y = 5x - 7 + 2x^{-1/2} + 3x^{-1}$

$\frac{dy}{dx} = 5 + (2)(-\frac{1}{2})x^{-3/2} + (3)(-1)x^{-2}$

$\frac{dy}{dx} = 5 - x^{-3/2} - 3x^{-2}$

$= 5 - \frac{1}{\sqrt[3]{x^2}} - \frac{3}{x^2}$

For the curve $C$ with equation $y = f(x)$,

$$\frac{dy}{dx} = x^3 + 2x - 7.$$

(a) Find $\frac{d^2y}{dx^2}$.

$$\frac{d^2y}{dx^2} = 3x^2 + 2$$

**(2)**

(b) Show that $\frac{d^2y}{dx^2} \geq 2$ for all values of $x$.

$x^2$ is always positive

$x^2 \geq 0$

so $\frac{d^2y}{dx^2} \geq 0 + 2$

**(1)**

Given that the point $P(2, 4)$ lies on $C$,

(c) find $y$ in terms of $x$,

**(5)**

(d) find an equation for the normal to $C$ at $P$ in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**(5)**

$$y = \int \frac{dy}{dx} = \int x^3 + 2x - 7$$

$$y = \frac{x^4}{4} + x^2 - 7x + c$$

$$y = \frac{x^4}{4} + x^2 - 7x + c$$

Put in point $(2,4)$

$$4 = \frac{2^4}{4} + 2^2 - 7(2) + c$$

$$4 = \frac{16}{4} + 4 - 14 + c$$

$$4 = 4 + 4 - 14 + c$$

$$4 = -6 + c$$

$$10 = c$$

$$y = \frac{x^4}{4} + x^2 - 7x + 10$$

Gradient at $P(2,4)$

$$\frac{dy}{dx} = x^3 + 2x - 7$$

$$= 8 + 4 - 7$$

$m = 5$ Tangent

$$y = -\frac{1}{5}x + c$$

$$4 = \left(-\frac{1}{5}\right)(2) + c$$

$$4 = -\frac{2}{5} + c$$

$$4\tfrac{2}{5} = c$$

$$y = -\tfrac{1}{5}x + \tfrac{22}{5}$$

$$5x - y - 22 = 0$$

$y = x^2 - k\sqrt{x}$, where $k$ is a constant.

(a) Find $\frac{dy}{dx}$.

**(2)**

(b) Given that $y$ is decreasing at $x = 4$, find the set of possible values of $k$.

**(2)**

$y = x^2 - k x^{1/2}$

$\frac{dy}{dx} = 2x - \frac{1}{2} k x^{-1/2}$

$\frac{dy}{dx} < 0$ if $y$ is decreasing

$2x - \frac{1}{2} k x^{-1/2} < 0$ when $x = 4$

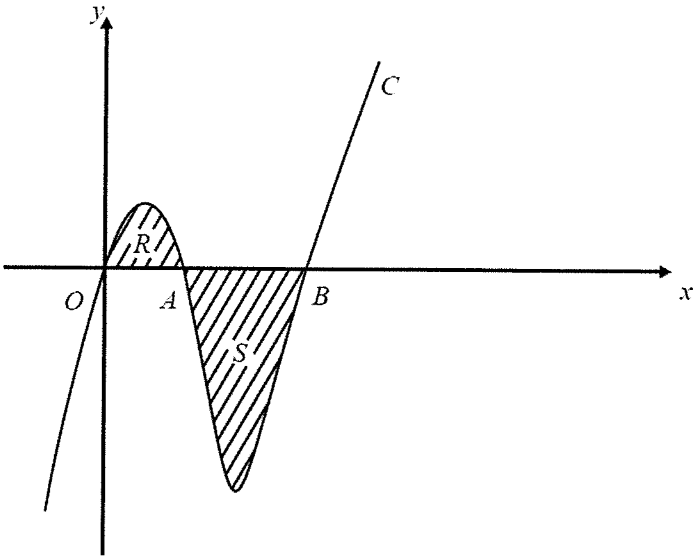
$8 - \frac{1}{2} k 4^{-1/2} < 0$

$8 - \frac{1}{4} k < 0$

$8 < \frac{1}{4} k$

$32 < k$

Ans $k > 32$




The diagram above shows part of the curve $C$ with equation $y = f(x)$, where

$$f(x) = x^3 - 6x^2 + 5x.$$

The curve crosses the $x$-axis at the origin $O$ and at the points $A$ and $B$.

(a) Factorise $f(x)$ completely. **(3)**

$f(x) = x(x^2 - 6x + 5)$
$= x(x-5)(x-1)$

(b) Write down the $x$-coordinates of the points $A$ and $B$. **(1)**

Answer $A = (1, 0)$ $B = (5, 0)$

(c) Find the gradient of $C$ at $A$. **(3)**

$f'(x) = 3x^2 - 12x + 5$
$f'(1) = 3(1) - 12(1) + 5 = -4$

The region $R$ is bounded by $C$ and the line $OA$, and the region $S$ is bounded by $C$ and the line $AB$.

(d) Use integration to find the area of the combined regions $R$ and $S$, shown shaded in the diagram above. **(7)**

$$\int_0^1 (x^3 - 6x^2 + 5x)\,dx + \int_1^5 (x^3 - 6x^2 + 5x)\,dx$$

$$\left[\frac{x^4}{4} - 2x^3 + \frac{5x^2}{2}\right]_0^1 + \left[\frac{x^4}{4} - 2x^3 + \frac{5x^2}{2}\right]_1^5$$

The function

$$f(x) = 2x^3 - 3x^2 - px + 6$$

has a turning point at $x = 2$

(i) Find the value of $p$. [3]

(ii) Find the range of values of $x$ for which $f(x)$ is an increasing function. [4]

$f(x) = 2x^3 - 3x^2 - px + 6$

$f'(x) = 6x^2 - 6x - p$

$f'(2) = 0$ Because turning point at $x = 2$

$0 = 6(2)^2 - 6(2) - p$

$0 = 24 - 12 - p$

$12 = p$

$f(x) = 2x^3 - 3x^2 - 12x + 6$

$f'(x) = 6x^2 - 6x - 12$

When is it increasing

$f'(x) > 0$

$6x^2 - 6x - 12 > 0$

$6(x^2 - x - 2) > 0$

$6(x-2)(x+1) > 0$

$\boxed{\text{solve using magic calculator } ax^2 + bx + c > 0}$

Ans $x < -1$ & $x > 2$

The curve $C$ has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\frac{dy}{dx}$

$$\frac{dy}{dx} = 12x^3 - 24x^2$$

(ii) $\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = 36x^2 - 48x$$

(3)

(b) Verify that $C$ has a stationary point when $x = 2$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

Check
Stationary point when $x = 2$

$$\frac{dy}{dx} = 0$$

$$12x^3 - 24x^2 = 0$$

$$12x^2(x-2) = 0$$

$$x = 0 \qquad x = 2$$

(c) When $x = 2$

$$\frac{d^2y}{dx^2} = 36(2)^2 - 48(2)$$

$$= 36(4) - 96$$

$$= 144 - 96$$

$$= 48$$

$\frac{d^2y}{dx^2} > 0$ so Minimum Turning Point

Given that $y = 2x^2 - \frac{6}{x^3}$, $x \neq 0$,

$y = 2x^2 - 6x^{-3}$

(a) find $\frac{dy}{dx}$,

$\frac{dy}{dx} = 4x + 18x^{-4}$

**(2)**

(b) evaluate $\int_1^3 y \, dx$.

**(4)**

$$\int_1^3 2x^2 - 6x^{-3} \, dx$$

$$= \left[\frac{2x^3}{3} - \frac{6x^{-2}}{-2}\right]_1^3$$

$$= \left[\frac{2x^3}{3} + \frac{3}{x^2}\right]^3 - \left[\frac{2x^3}{3} + \frac{3}{x^2}\right]_1$$

$$= \left[\frac{2 \times 27}{3} + \frac{3}{3^2}\right] - \left[\frac{2}{3} + 3\right]$$

$$= \left[\frac{54}{3} + \frac{1}{3}\right] - \left[\frac{2}{3} + 3\right]$$

$$= \frac{55}{3} - \frac{11}{3}$$

$$= \frac{44}{3}$$