Solve the simultaneous equations

$$x + y = 3$$
$$x^2 + 2y^2 - 8x = 6$$

Usual way to do is substitute one equation into the other

$x + y = 3$

$x = 3 - y$

so $x^2 + 2y^2 - 8x = 6$

$(3-y)^2 + 2y^2 - 8(3-y) = 6$

$9 - 6y + y^2 + 2y^2 - 24 + 8y - 6 = 0$

$3y^2 + 2y - 21 = 0$

$(3y - 7)(y + 3) = 0$

$y = \frac{7}{3}$ $\quad$ $y = -3$

then

$y = \frac{7}{3}$

$x = 3 - \frac{7}{3}$

$x = \frac{2}{3}$

then

$y = -3$

$x = 3 - (-3)$

$x = 6$

check

A circle C has equation

$$x^2 + y^2 - 6x + 8y - 75 = 0.$$

(a) Write down the coordinates of the centre of C, and calculate the radius of C. **(3)**

A second circle has centre at the point (15, 12) and radius 10.

clue

(b) Sketch both circles on a single diagram and find the coordinates of the point where they touch. **(4)**

$x^2 + y^2 - 6x + 8y - 75 = 0$

$x^2 - 6x \quad + y^2 + 8y \qquad = 75$

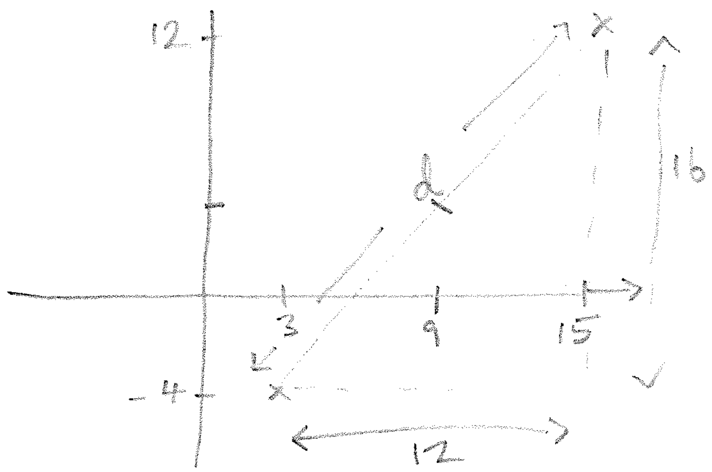
$(x-3)^2 - 9 + (y+4)^2 - 16 = 75$

$(x-3)^2 + (y+4)^2 = 100$

circle centre $(3, -4)$

radius $= 10$

$(x-15)^2 + (y-12)^2 = 100$



What is distance between centre

Pyth.

$d^2 = 12^2 + 16^2$

$d^2 = 144 + 256$

$d^2 = 400$

$d = 20$

so distance between centres is 20

so circles must just touch at the mid-point

$(9, 4)$

Two circles C_1 and C_2 have equations

$$(x-2)^2 + y^2 = 9 \text{ and } (x-5)^2 + y^2 = 9$$

respectively.

(a) For each of these circles state the radius and the coordinates of the centre. **(3)**

(b) Sketch the circles C_1 and C_2 on the same diagram. **(3)**

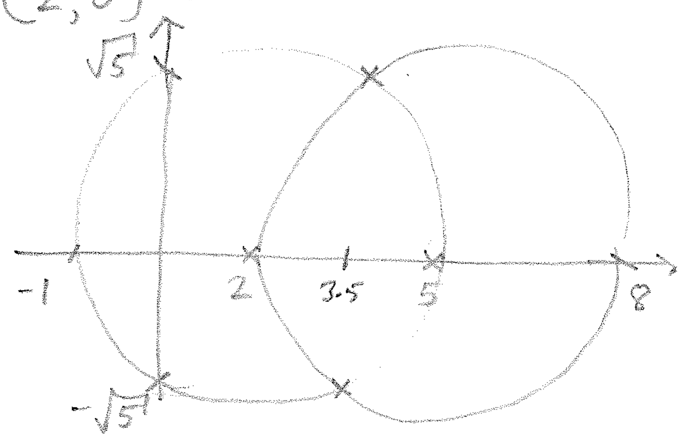
(c) Find the exact distance between the points of intersection of C_1 and C_2 **(3)**

a) $(x-2)^2 + y^2 = 9$

centre $(2,0)$ radius 3

$(x-5)^2 + y^2 = 9$

Centre $(5,0)$ Radius $= 3$



where does C_1 cut the y axes.

$(-2)^2 + y^2 = 9$

$y^2 = 5$

$y = \pm\sqrt{5}$

By symmetry points are (3.5, ?)

Put in $x = 3.5$ to get y

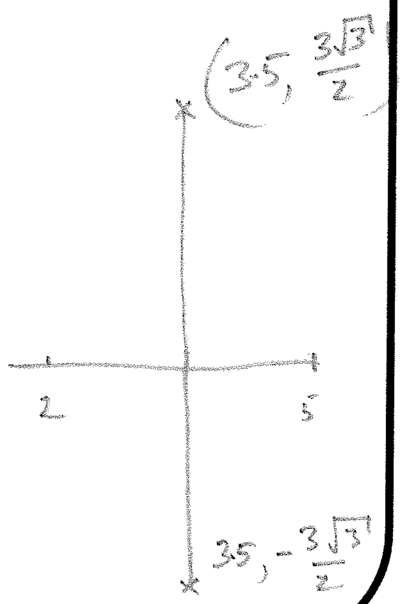
$(3.5-2)^2 + y^2 = 9$

$1.5^2 + y^2 = 9$

$y^2 = 9 - 2.25$

$y^2 = 6.75$

$y = \pm \frac{3\sqrt{3}}{2}$



Distance $= 2 \times \frac{3\sqrt{3}}{2} = 3\sqrt{3}$

Ans Exact $= 3\sqrt{3}$

A circle C has radius $\sqrt{5}$ and has its centre at the point with coordinates (4, 3).

(a) Prove that an equation of the circle C is $x^2 + y^2 - 8x - 6y + 20 = 0$.

(3)

The line l, with equation $y = 2x$, is a tangent to the circle C.

(b) Find the coordinates of the point where the line l touches C.

(4)

a)

$(x-4)^2 + (y-3)^2 = (\sqrt{5})^2$

$x^2 - 8x + 16 + y^2 - 6y + 9 = 5$

$x^2 - 8x + 16 + y^2 - 6y + 9 - 5 = 0$

$x^2 - 8x + y^2 - 6y + 20 = 0$

b)

Equat $y = 2x$ is tangent so

solve simultaneously circle and $y = 2x$

$x^2 - 8x + (2x)^2 - 6(2x) + 20 = 0$

$x^2 - 8x + 4x^2 - 12x + 20 = 0$

$5x^2 - 20x + 20 = 0$

$5(x^2 - 4x + 4) = 0$

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

so $x = 2$ repeated so only one x value

$y = 2(2)$ $y = 4$ Point $(2,4)$

check it is on the circle

$(x-4)^2 + (y-3)^2$

$(2-4)^2 + (4-3)^2$

$2^2 + 1^2$

$4 + 1 = 5$ ✓

Answer $(2,4)$

Solve the simultaneous equations

$$x + 2y = 3$$
$$x^2 + y^2 - 2xy = 6$$

$x + 2y = 3$

$x = 3 - 2y$

$x^2 + y^2 - 2xy = 6$

$(3-2y)^2 + y^2 - 2(3-2y)(y) = 6$

$9 - 12y + 4y^2 + y^2 - 6y + 4y^2 - 6 = 0$

$9y^2 - 18y + 3 = 0$

$3(3y - 6y + 1) = 0$

$3(3y \quad)(y \quad)$ Does not Factorise

using Equation Solver
or Quadratic Formula

$y = \frac{3+\sqrt{6}}{3}$ and $y = \frac{3-\sqrt{6}}{3}$

$x = 3 - 2y$

$x = 3 - 2\left(\frac{3+\sqrt{6}}{3}\right)$

$x = \frac{3-2\sqrt{6}}{3}$

$x = 3 - 2\left(\frac{3-\sqrt{6}}{3}\right)$

$x = \frac{9+2\sqrt{6}}{3}$

Answer

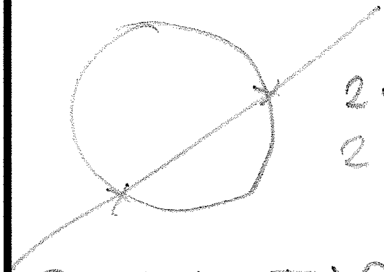
$x = \frac{3-2\sqrt{6}}{3}$ and $y = \frac{3+\sqrt{6}}{3}$

and $x = \frac{9+2\sqrt{6}}{3}$ and $y = \frac{3-\sqrt{6}}{3}$

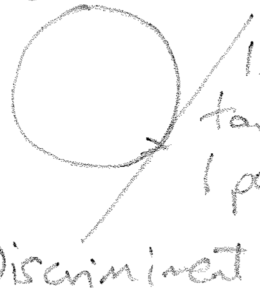
The line with equation $2x + 3y = k$ is a tangent to the circle with equation $x^2 + y^2 + 6x + 4y = 0$.
Find the two possible values of k.

only one solution

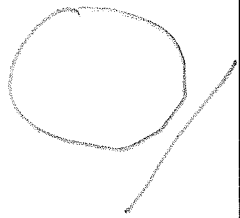
no solutions



2 solution
2 points



1 solution
tangent
1 point



Discriminent >0 Discriminent $=0$ Discriment <0

$2x + 3y = k$

$x^2 + y^2 + 6x + 4y = 0$

$2x+3y=k$

$x = \frac{k-3y}{2}$ put this into circle equation

$\left(\frac{k-3y}{2}\right)^2 + y^2 + 6\left(\frac{k-3y}{2}\right) + 4y = 0$

$\frac{k^2-6ky+9y^2}{4} + y^2 + 3k - 9y + 4y = 0$ ✓

multiply all by 4 to get rid of $\overline{\ \ 4\ \ }$

$k^2 - 6ky + 9y^2 + 4y^2 + 12k - 36y + 16y = 0$

$13y^2 + (-20-6k)y + (k^2+12k) = 0$

$a = 13 \quad b = -20-6k \quad c = k^2+12k$

$b^2 - 4ac = 0$

$(-20-6k)^2 - 4(13)(k^2+12k) = 0$

$400 + 240k + 36k^2 - 52k^2 - 624k = 0$

$-16k^2 - 384k + 400 = 0$

solving $k=1$ $\quad k=-25$

The line with equation $y = 1 - x$ intersects the circle with equation $x^2 + y^2 + 6x + 2y = 27$ at the points A and B.

Find the length of the chord AB, giving your answer in the form $k\sqrt{2}$.

Solve Putting $y=1-x$ into circle

$$x^2 + (1-x)^2 + 6x + 2(1-x) = 27$$

$$x^2 + 1 - 2x + x^2 + 6x + 2 - 2x - 27 = 0$$

$$2x^2 + 2x - 24 = 0$$

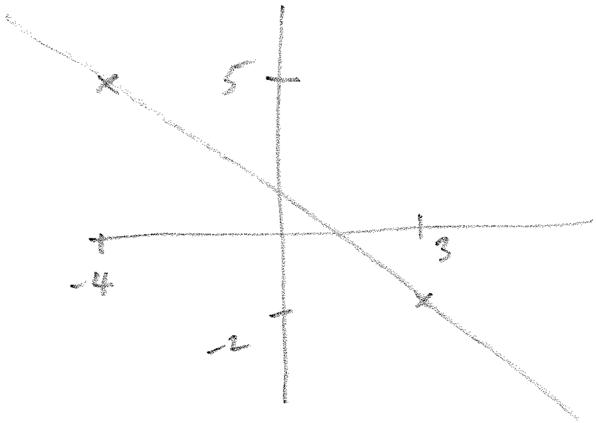
$$3(x^2 + x - 12) = 0$$

$$3(x+4)(x-3) = 0$$

$$x = -4 \quad \& \quad x = 3$$

Points using $y = 1 - x$.

$(-4, 5)$ $(3, -2)$



Distance using Pyth

$$\sqrt{7^2 + 7^2}$$

$$\sqrt{49 + 49}$$

$$\sqrt{98}$$

$$= \sqrt{49}\sqrt{2}$$

$$= 7\sqrt{2}$$

Ans Chord is $7\sqrt{2}$

Find in each case the coordinates of the points where the line l intersects the circle C.

a $l : y = x - 4$ $\quad C : x^2 + y^2 = 10$

b $l : 3x + y = 17$ $\quad C : x^2 + y^2 - 4x - 2y - 15 = 0$

c $l : y = 2x + 2$ $\quad C : 4x^2 + 4y^2 + 4x - 8y - 15 = 0$

a) $y = x - 4$ Sub in $\quad x^2 + (x-4)^2 = 10$

$$x^2 + x^2 - 8x + 16 = 10$$

$$2x^2 - 8x + 6 = 0$$

$$2(x^2 - 4x + 3) = 0$$

$$2(x-3)(x-1) = 0$$

$$x = 3 \qquad x = 1$$

$$(3,-1) \quad \& \quad (1,-3)$$

b) $3x + y = 17$

$y = 17 - 3x$

$$x^2 + (17-3x)^2 - 4x - 2(17-3x) - 15 = 0$$

$$x^2 + 289 - 102x + 9x^2 - 4x - 34 + 6x - 15 = 0$$

$$10x^2 - 100x + 250 = 0$$

$$10(x^2 - 10x + 25) = 0$$

$$10(x-5)(x-5)$$

$$x = 5$$

Point $(5, 2)$

$y = 2x + 2$

$$4x^2 + 4y^2 + 4x - 8y - 15 = 0$$

$$4x^2 + 4(2x+2)^2 + 4x - 8(2x+2) - 15 = 0$$

$$4x^2 + 16x^2 + 32x + 16 + 4x - 16x - 16 - 15 = 0$$

$$20x^2 + 20x - 15 = 0$$

$$5(4x + 4x - 3) = 0$$

$$5(2x+3)(2x-1) = 0$$

$$x = -3/2 \qquad x = 1/2$$

$$(-3/2, -1) \qquad (1/2, 3)$$

The circle C has equation $x^2 + y^2 - 4x - 6 = 0$ and the line l has equation $y = 3x - 6$.

a Show that l passes through the centre of C. **(3)**

b Find an equation for each tangent to C that is parallel to l. **(6)**

$x^2 + y^2 - 4x - 6 = 0$ complete the square

$(x-2)^2 - 4 + y^2 - 6 = 0$

$(x-2)^2 + y^2 = 10$

centre $(2,0)$ Radius $\sqrt{10}$

Does point $(2,0)$ lie on line $y = 3x - 6$

Substitute in to check

$0 = 3(2) - 6$

$0 = 6 - 6$ True. Yes it does pass through centre

$0 = 0$

Parallel to $y = 3x - 6$

means $y = 3x + k$

Solve simultaneously

$y = 3x + k$ and $x^2 + y^2 - 4x - 6 = 0$

$x^2 + (3x+k)^2 - 4x - 6 = 0$

$x^2 + 9x^2 + 6kx + k^2 - 4x - 6 = 0$

$10x^2 + (6k-4)x + (k^2 - 6) = 0$

Tangent means Discriminent $= 0$

$a = 10$ $b = 6k - 4$ $c = k^2 - 6$

$b^2 - 4ac = 0$

$(6k-4)^2 - 4(10)(k^2 - 6) = 0$

$36k^2 - 48k + 16 - 40k^2 + 240 = 0$

$-4k^2 + 48k + 256 = 0$

$-4(k^2 + 12k - 64) = 0$

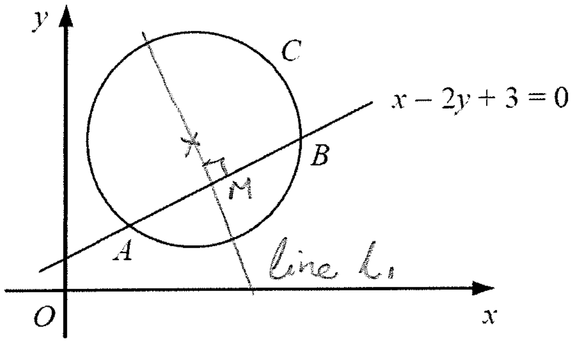
$-4(k-4)(k+16) = 0$

$k = 4$ & -16

Lines

$y = 3x + 4$

$y = 3x - 16$



$y = \frac{1}{2}x + \frac{3}{2}$

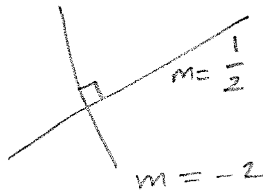
The line with equation $x - 2y + 3 = 0$ intersects the circle C at the points A and B as shown in the diagram above. Given that the centre of C has coordinates (6, 7),

a find the coordinates of the mid-point of the chord AB. **(6)**

Given also that the x-coordinate of the point A is 3,

b find the coordinates of the point B, **(3)**

c find an equation for C. **(2)**



$(x-6)^2 + (y-7)^2 = r^2$

AB mid-point = ? = M

line l_1 $y = mx + c$

$y = -2x + c$

$7 = -2(6) + c$

$7 = -12 + c$

$19 = c$

Put in (6,7) to get c

Where do lines cross?

Solve simultaneously

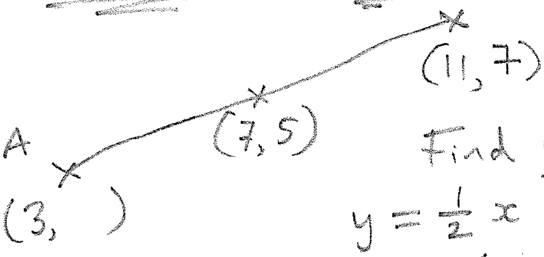
$y = \frac{1}{2}x + 3/2$ & $y = -2x + 19$

$\frac{1}{2}x + 3/2 = -2x + 19$

$2\frac{1}{2}x = 17\frac{1}{2}$

$x = 7$

Mid-point is (7, 5)



Find y from line

$y = \frac{1}{2}x + 3/2 \longrightarrow y = 7$

Ans $B = (11, 7)$

Centre (6,7) $B = (11, 7)$ Radius = 5

Circle $(x-6)^2 + y-7)^2 = 5^2$

The circle C has equation $x^2 + y^2 - 6x - 12y + 28 = 0$.

a Find the coordinates of the centre of C. **(2)**

The line $y = x - 2$ intersects C at the points A and B.

b Find the length AB in the form $k\sqrt{2}$. **(6)**

Complete the square

$x^2 - 6x + y^2 - 12y + 28 = 0$

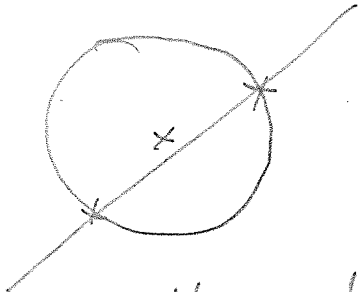
$(x-3)^2 - 9 + (y-6)^2 - 36 + 28 = 0$

$(x-3)^2 + (y-6)^2 = 17$

Centre $(3,6)$

Radius $\sqrt{17}$

Lines $y = x - 2$



Line crosses twice

$y = x - 2$

$x^2 + (x-2)^2 - 6x - 12(x-2) + 28 = 0$

Solve simultaneously

$x^2 + x^2 - 4x + 4 - 6x - 12x + 24 + 28 = 0$

$2x^2 - 22x + 56 = 0$

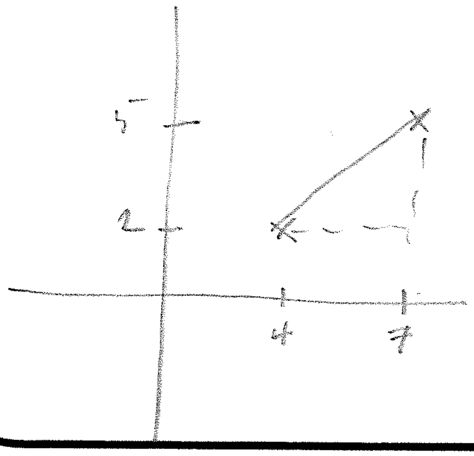
$2(x^2 - 11x + 28) = 0$

$2(x-4)(x-7) = 0$

$x = 4 \qquad x = 7$

Points $(4,2)$ and $(7,5)$ by using $y = x - 2$

$A = (4,2) \qquad B(7,5)$



Distance by Pythagoras'

$\sqrt{3^2 + 3^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= \sqrt{9} \times \sqrt{2}$

Ans $3\sqrt{2}$

The circle C has the equation $x^2 + y^2 + 2x - 14y + 30 = 0$.

a Find the coordinates of the centre of C. **(2)**

b Find the radius of C, giving your answer in the form $k\sqrt{5}$. **(2)**

c Show that the line $y = 2x - 1$ is a tangent to C and find the coordinates of the point of contact. **(4)**

$x^2 + 2x + y^2 - 14y + 30 = 0$

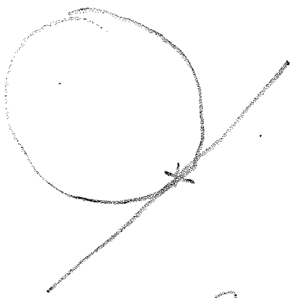
$(x+1)^2 - 1 + (y-7)^2 - 49 + 30 = 0$

$(x+1)^2 + (y-7)^2 = 20$

Centre $(-1, 7)$ Radius $= \sqrt{20} = 2\sqrt{5}$

Show $y = 2x - 1$

only has 1 point
or 1 solution



Solve $y = 2x - 1$ & circle simultaneously

$x^2 + (2x-1)^2 + 2x - 14(2x-1) + 30 = 0$

$x^2 + 4x^2 - 4x + 1 + 2x - 28x + 14 + 30 = 0$

$5x^2 - 30x + 45 = 0$

$5(x^2 - 6x + 9) = 0$

$5(x-3)(x-3) = 0$

Repeated Root when $x = 3$

Point $(3, 5)$ using $y = 2x - 1$

Ans point of contact $= (3, 5)$