Find the value of the constant p such that the equation $x^2 + x + p = 0$ has equal roots.

Discriminent $b^2 - 4ac$

$b^2 - 4ac > 0$ 2 different real roots or solutions

$b^2 - 4ac = 0$ 2 equal roots

$b^2 - 4ac < 0$ no roots

$x^2 + x + p = 0$

$a = 1 \quad b = 1 \quad c = p$

$b^2 - 4ac = 0$

$1^2 - 4(1)(p) = 0$

$1 = 4p$

$\frac{1}{4} = p$

Ans $\frac{1}{4}$

Check on your fancy calculator

$\boxed{\begin{matrix} x\ y \\ =0 \end{matrix}}$ A: ~~EQUATION~~/FUNC

Given that $q \neq 0$, find the value of the constant q such that the equation $x^2 + 2qx - q = 0$ has a repeated root.

$x^2 + 2qx - q = 0$

Repeated root means $b^2 - 4ac = 0$

$a = 1 \quad b = 2q \quad c = -q$

$b^2 - 4ac = 0$

$(2q)^2 - 4(1)(-q) = 0$

$4q^2 + 4q = 0$

$4q(q+1) = 0$

$q = 0$ but not possible $\quad q = -1$

Ans $q = -1$

Check with white calculator

$x^2 - 2x + 1 = 0$

gives

$(x-1)(x-1)$

repeated root $x = 1$

The equation $x^2 + kx + 2 = 0$, where k is a constant has no real roots.

Find the set of possible values for k.

Discriminent < 0 $\qquad a = 1 \quad b = k \quad c = 2$

$b^2 - 4ac < 0$

$k^2 - 4(1)(2) < 0$

$k^2 - 8 < 0$

$k^2 < 8$

$-4 < k < +4$

Ans $-4 < k < +4$

Check on white calculator.

The equation $(k+5)x^2 + 4x + (k+2) = 0$, where k is a constant has two distinct real solutions for x.

Find the set of possible values for k.

2 distinct real roots

$b^2 - 4ac > 0$

$\boxed{\begin{aligned} a &= k+5 \\ b &= 4 \\ c &= k+2 \end{aligned}}$

$(4)^2 - 4(k+5)(k+2) > 0$

$16 - 4(k^2 + 7k + 10) > 0$

$16 - 4k^2 - 28k - 40 > 0$

$-4k^2 - 28k - 24 > 0$

$0 > 4k^2 + 28k + 24$

Solve using calculator $4k^2 + 28k + 24 < 0$

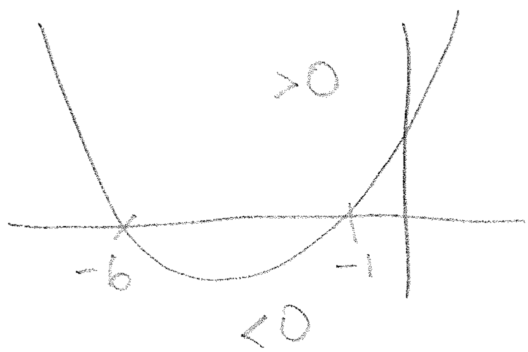
or $4k^2 + 28k + 24 < 0$

$4(k^2 + 7k + 6) < 0$

$4(k+6)(k+1) < 0$

$k = -6 \quad k = -1$

$4k^2$ means ∪ happy graph



$4(k+6)(k+1) < 0$

Ans

$-6 < k < -1$

Given that the x-axis is a tangent to the curve with the equation

$$y = x^2 + rx - 2x + 4,$$

find the two possible values of the constant r.

x axis is also $y=0$ line

so solve $y=0$ and $y = x^2 + rx - 2x + 4$

simultaneously

$$0 = x^2 + rx - 2x + 4$$

$$0 = x^2 + (r-2)x + 4$$

But $y=0$ is a TANGENT so only one root

$$x^2 + (r-2)x + 4 = 0$$

Discriminent $= 0$

$a = 1$

$b = r - 2$

$c = 4$

$$b^2 - 4ac = 0$$

$$(r-2)^2 - 4(1)(4) = 0$$

$$r^2 - 4r + 4 - 16 = 0$$

$$r^2 - 4r - 12 = 0$$

$$(r-6)(r+2) = 0$$

$r = 6$ $\quad$ $r = -2$

Ans $r = 6$ & $r = -2$

Check on calculator

$x^2 + 4x + 4$ & $x^2 - 4x + 4$

$(x+2)^2$ $\quad$ $(x-2)^2$

The curve with equation $y = px^2 - 4px - 5p$, where p is a constant does not intersect the line with equation $y = 2x - 12$.

(a) Show that $9p^2 - 8p + 1 < 0$

(b) Find the set of possible values for p.

$y = px^2 - 4px - 5p$

does not intersect $y = 2x - 12$

solving simultaneously does not give any possible answers

$px^2 - 4px - 5p = 2x - 12$

$px^2 - 4px - 2x - 5p + 12 = 0$

$px^2 + (-4p-2)x + (12-5p) = 0$

Discriminant < 0

$b^2 - 4ac < 0$

$a = p$

$b = -4p - 2$

$c = 12 - 5p$

$(-4p-2)^2 - 4(p)(12-5p) < 0$

$(-4p-2)(-4p-2) - 4p(12-5p) < 0$

$16p^2 + 16p + 4 - 48p + 20p^2 < 0$

$36p^2 - 32p + 4 < 0$

$4(9p^2 - 8p + 1) < 0$

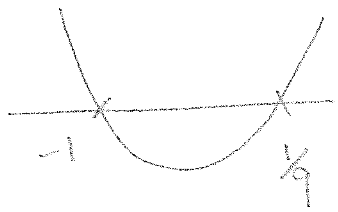
when is $9p^2 - 8p + 1 < 0$

solve using calculator

$9p^2 - 8p + 1 = 0$

$(9p-1)(p+1) = 0$

$p = 1/9 \quad p = -1$



Answer $-1 < p < 1/9$

The line $y = mx - 2$ is a tangent to the circle $x^2 + 6x + y^2 - 8y + 5 = 0$

Find the two possible values of m, giving your answers in exact form.

Solve simultaneously

$y = mx-2$ put this into the other equation

$x^2 + 6x + y^2 - 8y + 5 = 0$

$x^2 + 6x + (mx-2)^2 - 8(mx-2) + 5 = 0$

$x^2 + 6x + (mx-2)(mx-2) - 8(mx-2) + 5 = 0$

$x^2 + 6x + m^2x^2 - 4mx + 4 - 8mx + 16 + 5 = 0$

$x^2(1+m^2) + x(6-12m) + 25 = 0$

Discriminent $= 0$ because tangent

$b^2 - 4ac = 0$

$(6-12m)^2 - 4(1+m^2)(25) = 0$

$36 - 144m + 144m^2 - 100 - 100m^2 = 0$

$44m^2 - 144m - 64 = 0$

$4(11m^2 - 36m - 16) = 0$

$11m^2 - 36m - 16 = 0$

equation solver

$m = \frac{18 + 10\sqrt{5}}{11}$ & $\frac{18 - 10\sqrt{5}}{11}$

The line $y = mx + 2$ is a tangent to the circle $(x-5)^2 + (y+1)^2 = 15$

Find the two possible values of m, giving your answers in exact form.

Put in $y = mx + 2$ into other equation

$(x-5)^2 + (y+1)^2 = 15$

$(x-5)^2 + (mx+3)^2 = 15$

$x^2 - 10x + 25 + m^2x^2 + 6mx + 9 = 15$

$x^2(1+m^2) + x(6m-10) + 19 = 0$

Discriminant $= 0$

$a = 1+m^2$ $\quad b = 6m-10$ $\quad c = 19$

$b^2 - 4ac = 0$

$(6m-10)^2 - 4(1+m^2)(19) = 0$

$36m^2 - 120m + 100 - 76 - 76m^2 = 0$

$-40m^2 - 120m + 24 = 0$

$40m^2 + 120m - 24 = 0$

$4(10m^2 + 30m - 6) = 0$

$\frac{-15 + \sqrt{285}}{10}$ & $\frac{-15 - \sqrt{285}}{10}$

The equation $x^2 + (n+1)x + (3-3n) = 0$, where n is a constant has two distinct real roots.

Find the set of possible values for n.

Discriminant > 0

$a = 1 \quad b = n+1 \quad c = 3-3n$

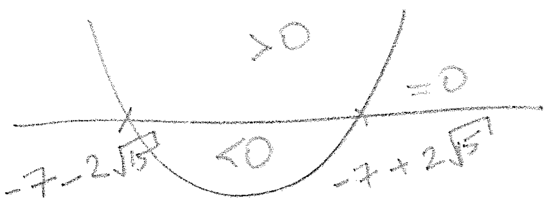
$b^2 - 4ac > 0$

$(n+1)^2 - 4(1)(3-3n) > 0$

$n^2 + 2n + 1 - 12 + 12n > 0$

$n^2 + 14n - 11 > 0$

$n = -7 + 2\sqrt{15}$ & $-7 - 2\sqrt{15}$



Ans > 0

when $x < -7 - 2\sqrt{15}$

& $x > -7 + 2\sqrt{15}$

The equation $kx^2 + 6kx + 2 = 0$, where k is a constant has no real roots.

Find the set of possible values for k.

Discriminant < 0

$b^2 - 4ac < 0$

$a = k$

$b = 6k$

$c = 2$

$(6k)^2 - 4(k)(2) < 0$

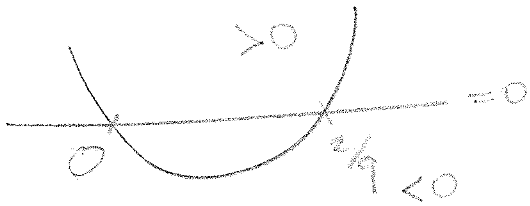
$36k^2 - 8k < 0$

$4(9k^2 - 2k) < 0$

solve $36k^2 - 8k = 0$

$4k(9k - 2) = 0$

$k = 0 \qquad k = 2/9$



Ans $36k^2 - 8k < 0$

when $0 < k < 2/9$