The population of a species of plant in a field is modelled using the formula $P = 50e^{0.1t}$
Where t is the number of weeks since the population was first recorded.

(a) Write down the number of the plants when the population was first recorded. when $t=0$ **(1)**

(b) ~~Find the rate of increase in the population 10 weeks after the population was first recorded.~~ **(2)**

(c) Find how many weeks it takes for the number of plants to exceed 300. **(4)**

a) $P = 50e^{0.1t}$

when $t=0$ $\quad P = 50e^{0.1(0)}$

$P = 50 \times 1$

$P = 50$ $\quad$ Ans 50 plants

b) NOT on AS specification

c) exceed 300

$300 = 50e^{0.1t}$

$6 = e^{0.1t}$

$\ln(6) = \ln(e^{0.1t})$

$\ln(6) = 0.1t$

$17.9 = t$ $\quad$ Ans 18 weeks.

check by putting in $t=17$ & $t=18$

$t=17$

$P = 50e^{0.1(17)}$

$P = 273$ ×

$t=18$

$P = 50e^{0.1(18)}$

$P = 302$ ✓

Ans 18 weeks!

The decay of a radioactive substance is modelled using the formula $N = 1000e^{-kt}$
Where N is the number of atoms after t years and k is a positive constant.

(a) Write down the number of atoms when the substance started to decay. **(1)**

Given it takes 14.4 years for half of the substance to decay.

(b) Find the value of k to three significant figures. **(4)**

(c) Calculate the number of atoms left when t=30. **(1)**

(a) when $t=0$ $\quad N = 1000e^{-k(0)} = 1000$

Ans 1000 atoms

(b) $t = 14.4$ for decay to 500

$$500 = 1000e^{-k(14.4)}$$

$$\frac{500}{1000} = e^{-k(14.4)}$$

$$\ln\left(\frac{500}{1000}\right) = -k(14.4)$$

Ans $k = 0.0481$

(c) when $t = 30$

$$N = 1000e^{-0.0481t}$$

when $t=30$

$$N = 1000e^{-0.0481(30)}$$

$$N = 236 \text{ atoms}$$

Find the set of values of x for which

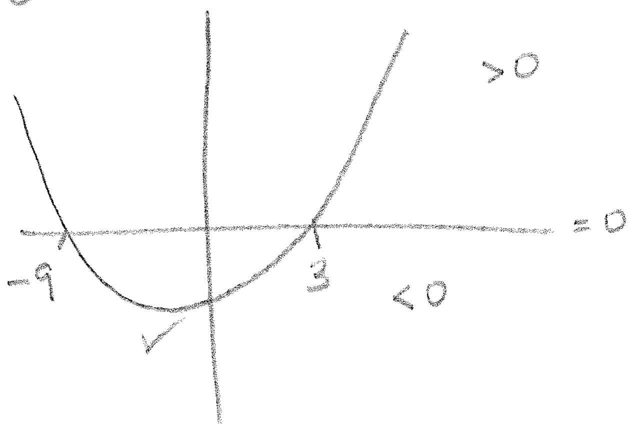
$$(x + 5)(x + 1) < 32$$

$x^2 + 6x + 5 < 32$

$x^2 + 6x - 27 < 0$

$(x + 9)(x - 3) < 0$

$x = -9 \qquad x = 3$



Ans $-9 < x < +3$

Solve the inequality

$$x(x + 1) \leq 12$$

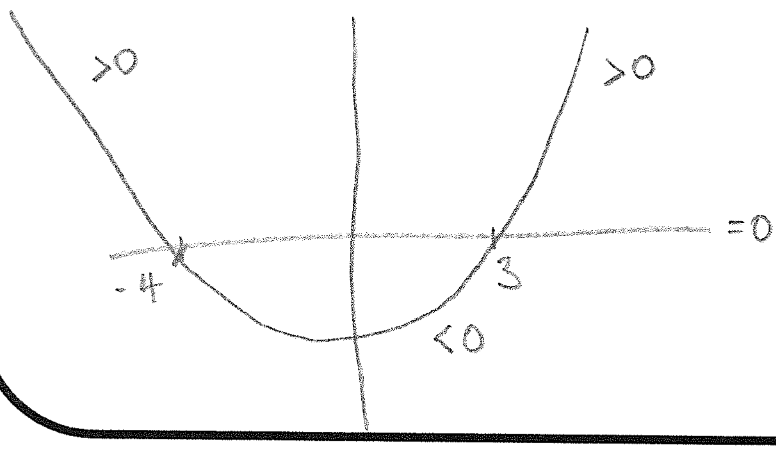
$x(x+1) \leqslant 12$

$x^2 + x \leqslant 12$

$x^2 + x - 12 \leqslant 0$

$(x + 4)(x - 3) \leqslant 0$

$x = -4 \qquad x = 3$



Ans

$-4 \leqslant x \leqslant 3$

Check

$2(13 + 2x) < (6 + x)(1 - x)$

$26 + 4x < 6 - 6x + x - x^2$

$x^2 + 9x + 20 < 0$

You can do all this now on your calculator

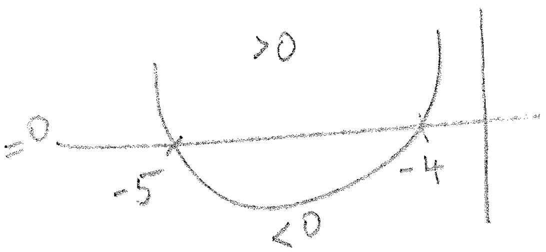
But method is

$x^2 + 9x + 20 = 0$

$(x+5)(x+4) = 0$

$x = -5$ $\quad$ $x = -4$

x^2 is $\cup$ graph



$x^2 + 9x + 20 < 0$

so below the axis

Ans $-5 < x < -4$

The temperature of water in a kettle is modelled using the formula $T = 75e^{-kt} + 22$

Where T is the temperature t minutes after the kettle is turned off and k is a positive constant.

~~(a) Find the rate of change of the temperature in terms of k~~ Not on AS Maths **(2)**

After 5 minutes the temperature of the water is 70°C

(b) Find the value of k **(3)**

(c) Find how many minutes it takes for the water to cool to 55°C **(4)**

$T = 75e^{-kt} + 22$

When $t=5$ then $T = 70$

$70 = 75e^{-5k} + 22$

$48 = 75e^{-5k}$

$\frac{48}{75} = e^{-5k}$

$\ln\left(\frac{48}{75}\right) = \ln\left(e^{-5k}\right)$

$-0.4463 = -5k$

$0.0893 = k$

$T = 75e^{-0.0893t} + 22$

$55 = 75e^{-0.0893t} + 22$

$33 = 75e^{-0.0893t}$

$\frac{33}{75} = e^{-0.0893t}$

$\ln\left(\frac{33}{75}\right) = -0.0893t$

$9.197 = t$

Ans 9.198 9.20 minutes

A quantity N is decreasing such that at time t

$$N = N_0 e^{kt}.$$

Given that at time $t = 10$, $N = 300$ and that at time $t = 20$, $N = 225$, find

a the values of the constants N_0 and k,

b the value of t when $N = 150$.

$\boxed{N = N_0 e^{kt}}$

$t = 10 \quad N = 300$

$300 = N_0 e^{10k}$

$\frac{300}{e^{10k}} = N_0$

$t = 20 \quad N = 225$

$225 = N_0 e^{20k}$

$225 = \frac{300}{e^{10k}} \cdot e^{20k}$

$225 e^{10k} = 300 e^{20k}$

$225 = 300 e^{10k}$

$\frac{225}{300} = e^{10k}$

$\ln\left(\frac{225}{300}\right) = 10k$

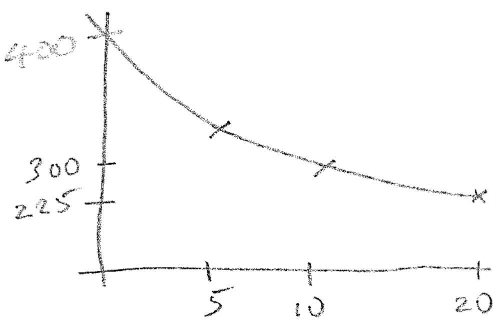
$-0.0288 = k \quad$ (3 sig. fig.)

Negative be

$N_0 = \frac{300}{e^{10(-0.0288)}}$

$N_0 = 400$

Ans $N_0 = 400 \quad k = -0.0288$

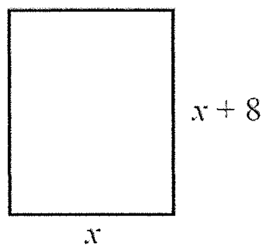


$150 = 400 e^{-0.0288t}$

$\frac{3}{8} = e^{-0.0288t}$

$34.1 = t$

Answer $t = 34.1$



The diagram shows a rectangular birthday card which is x cm wide and $(x + 8)$ cm tall.

Given that the height of the card is to be at least 50% more than its width,

a show that $x \leq 16$.

Given also that the area of the front of the card is to be at least 180 cm^2,

b find the set of possible values of x.

$x + 8 \geqslant 1.5(x)$

$x + 8 \geqslant 1.5x$

$8 \geqslant 1.5x - x$

$8 \geqslant 0.5x$

$16 \geqslant x$ Proved $16 \geqslant x$ or $x \leqslant 16$

Area $x(x+8) \geqslant 180$

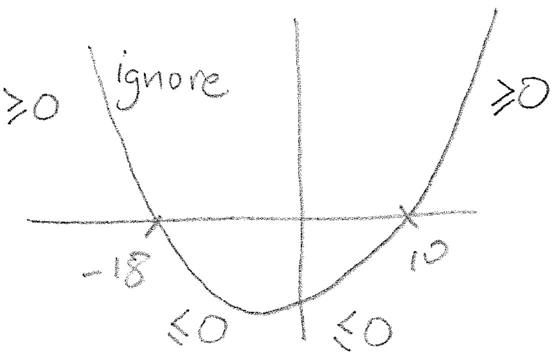
$x^2 + 8x \geqslant 180$

$x^2 + 8x - 180 \geqslant 0$

$x^2 + 8x - 180 = 0$

$(x+18)(x-10) = 0$

$x = -18$ $x = +10$



Area means $x \geqslant 10$

Length means $x \leqslant 16$

Possible values $10 \leqslant x \leqslant 16$

Given that $x - y = 8$,

and that $xy \leq 240$,

find the maximum value of $(x + y)$.

$\boxed{y = x - 8}$

$xy \leq 240$

$x(x - 8) \leq 240$

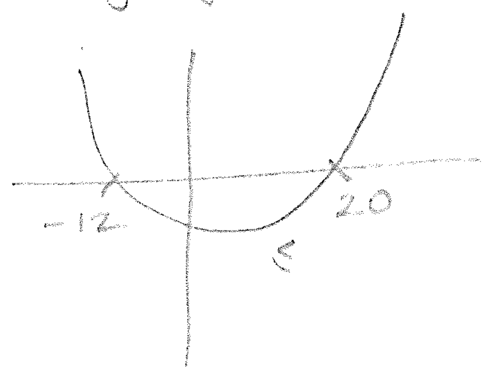
$x^2 - 8x \leq 240$

$x^2 - 8x - 240 \leq 0$

$(x - 20)(x + 12) \leq 0$

$x = 20 \quad x = -12$

Always get 0 on RHS



x values can be

$-12 \leq x \leq 20$

$y = x - 8$

$y = -12 - 8$

$y = -20$

$y = 20 - 8$

$y = 12$

Maximum Value is max x + max y

20 12

Maximum value $x + y$

$20 + 12$

32

A bead is projected vertically upwards in a jar of liquid with a velocity of 13 m s^{-1}.
Its velocity, v m s^{-1}, at time t seconds after projection, is given by

$$v = ce^{-kt} - 2.$$

$v = 13$
when $t = 0$

a Find the value of c. **(2)**

Given that the bead has a velocity of 7 m s^{-1} after 5.1 seconds, find

b the value of k correct to 4 decimal places, **(3)**

c the time taken for its velocity to decrease from 10 m s^{-1} to 4 m s^{-1}. **(5)**

$v = c e^{-kt} - 2$

$v = 13$ when $t = 0$

$13 = c e^{-k(0)} - 2$

$15 = c(1)$ Ans $c = 15$

when $t = 5.1$ $v = 7$

$7 = 15 e^{-5.1k} - 2$

$9 = 15 e^{-5.1k}$

$\frac{9}{15} = e^{-5.1k}$

$\ln\left(\frac{9}{15}\right) = -5.1k$ $k = 0.100$ (3 sig. fig.)

$10 = 15 e^{-0.100t} - 2$

$12 = 15 e^{-0.100t}$

$\frac{12}{15} = e^{-0.100t}$

$t = 2.23$ secs

$4 = 15 e^{-0.100t} - 2$

$6 = 15 e^{-0.100t}$

$\frac{6}{15} = e^{-0.100t}$

$t = 9.16$ secs

Time taken $= 9.16 - 2.23$
$= 6.93$ secs.

A colony of fast-breeding fish is introduced into a large, newly-built pond. The number of fish in the pond, n, after t weeks is modelled by

$$n = \frac{18000}{1+8c^{-t}}.$$

t weeks

a Find the initial number of fish in the pond. **(2)**

Given that there are 3600 fish in the pond after 3 weeks, use this model to

b show that $c = \sqrt[3]{2}$, **(3)**

$t = 3$

c find the time taken for the initial population of fish to double in size, giving your answer to the nearest day. **(4)**

$$n = \frac{18000}{1+8c^{-t}}$$

when $t = 0$ initially $\quad n = \frac{18000}{1+8c^{-0}} = \frac{18000}{9}$

$n = 2000$ fish initially

$$3600 = \frac{18000}{1+8c^{-3}}$$

$$1+8c^{-3} = \frac{18000}{3600}$$

$$\frac{8}{c^3} = 5-1$$

$$\frac{8}{c^3} = 4$$

$$2 = c^3$$

$$\sqrt[3]{2} = c$$

Double in size

$$4000 = \frac{18000}{1+8(\sqrt[3]{2})^{-t}}$$

$$1+8(\sqrt[3]{2})^{-t} = 4.5$$

$$8(\sqrt[3]{2})^{-t} = 3.5$$

$$\frac{8}{\sqrt[3]{2}^{\,t}} = 3.5$$

$$\frac{8}{3.5} = (\sqrt[3]{2})^t$$

$$\frac{16}{7} = 2^{t/3}$$

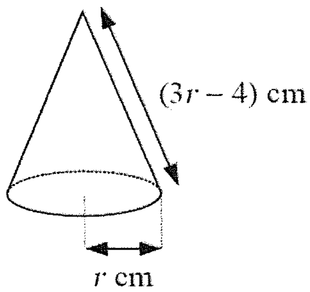
$$\ln\left(\frac{16}{7}\right) = \ln 2^{t/3}$$

$$\ln\left(\frac{16}{7}\right) = \frac{t}{3}(\ln 2)$$

$$3.578 = t$$

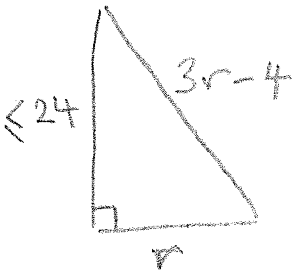
check it!

Ans 3 weeks 4 days



A party hat is designed in the shape of a right circular cone of base radius r cm and slant height $(3r-4)$ cm.

Given that the height of the cone must not be more than 24 cm, find the maximum value of r.



Pyth.

$(3r-4)^2 - r^2 \leq 24^2$

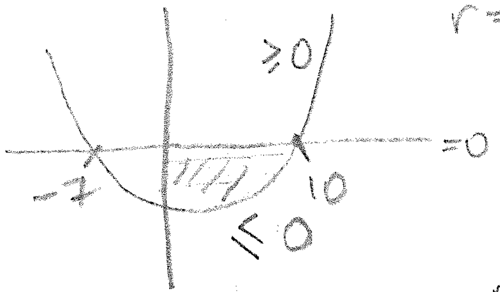
$9r^2 - 24r + 16 - r^2 \leq 576$

$8r^2 - 24r - 560 \leq 0$

$8(r^2 - 3r - 70) \leq 0$

$8(r-10)(r+7) \leq 0$

$r = 10 \qquad r = -7$



r values

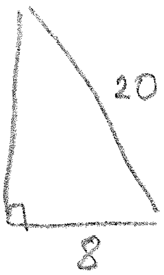
$0 < r \leq 10$

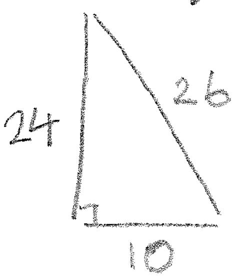
Ans $\underline{r = 10}$

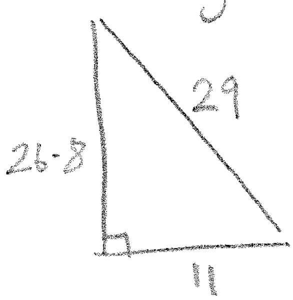
check it!

$r = 8$ $\qquad$ $r = 10$ ✓ $\qquad$ $r = 11$ wrong

True







Find the set of values of x for which

$$(3x-1)^2 < 5x-1.$$

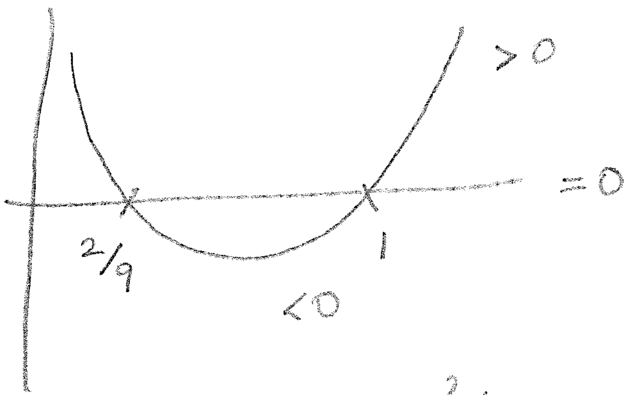
$(3x-1)(3x-1) < 5x-1$

$9x^2-6x+1 < 5x-1$

$9x^2-11x+2 < 0$

$(x-1)(9x-2)=0$

$x=1 \qquad x = 2/9$



Answer $\quad 2/9 < x < 1$

Check using fancy new calculator

$\boxed{\begin{matrix} x\ y \\ >0 \end{matrix}}$ B: INEQUALITY button

Select 2 degree

1. $ax^2+bx+c > 0$
2. < 0
3. ≥ 0
4. ≤ 0