Indices or Logs

$$49^{x+1} = \sqrt{7}$$

49 and 7 are connected so use indices

$$(7^2)^{x+1} = 7^{1/2}$$

$$7^{2x+2} = 7^{1/2}$$

then

$$2x+2 = 1/2$$

$$2x = -3/2$$

$$x = -3/4$$

CHECK IT!

$$\left(\tfrac{1}{6}\right)^{x+3} = 36$$

6 and 36 are connected with indices

$$\left(\frac{1}{6}\right)^{x+3} = 6^2$$

$$(6^{-1})^{x+3} = 6^2$$

$$6^{-x-3} = 6^2$$

$$-x-3 = 2$$

$$x+3 = -2$$

$$x = -5$$

CHECK IT!

$$\left(\frac{1}{6}\right)^{-2} \checkmark$$

$$\left(\tfrac{1}{2}\right)^{3x-1} = 8$$

2 and 8 are connected

$$(2^{-1})^{3x-1} = 2^3$$

$$2^{1-3x} = 2^3$$

$$1-3x = 3$$

$$-2 = 3x$$

$$-2/3 = x$$

CHECK IT!

a Find the value of t such that

$$\left(\tfrac{1}{4}\right)^{t-3} = 8.$$

b Solve the equation

$$\left(\tfrac{1}{3}\right)^{y} = 27^{y+1}.$$

(a) 4 and 8 are connected with indices
Connection is with 2 because

$2^2 = 4$ $\quad$ $2^3 = 8$

$$\left(\frac{1}{4}\right)^{t-3} = 8$$

$$(2^{-2})^{t-3} = 2^3$$

$$2^{6-2t} = 2^3$$

$$6 - 2t = 3$$
$$3 = 2t$$
$$3/2 = t$$

Check it!

(b)

$$\left(\frac{1}{3}\right)^y = 27^{y+1}$$

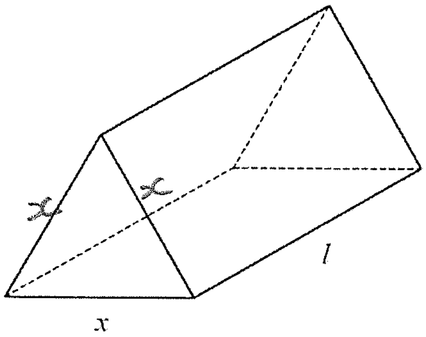
3 and 27 are connected

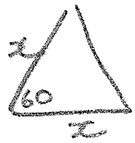
$$(3^{-1})^y = (3^3)^{y+1}$$

$$3^{-y} = 3^{3y+3}$$

$$-y = 3y + 3$$
$$-3 = 4y$$
$$-3/4 = y$$

Check it!





Area $= \frac{1}{2}ab\sin C$
$= \frac{1}{2}x^2\sin 60$
$= \frac{1}{2}\cdot x^2\cdot \sqrt{3}/2$
$= \frac{\sqrt{3}}{4}x^2$

The diagram shows a solid triangular prism. The cross-section of the prism is an equilateral triangle of side x cm and the length of the prism is l cm.

Given that the volume of the prism is 250 cm^3,

a find an expression for l in terms of x,

b show that the surface area of the prism, A cm^2, is given by

$$A = \frac{\sqrt{3}}{2}\left(x^2 + \frac{2000}{x}\right).$$

Given that x can vary,

c find the value of x for which A is a minimum, $\frac{dA}{dx} = 0$

d find the minimum value of A in the form $k\sqrt{3}$,

e justify that the value you have found is a minimum.

(a) Volume = Area $\times\, l$

$250 = \frac{\sqrt{3}}{4}x^2\cdot l$

$\frac{1000}{\sqrt{3}x^2} = l$

(b) Surface Area
= 2 ends + 3 rectangles
$= 2\left(\frac{\sqrt{3}}{4}x^2\right) + 3xl$
$= \frac{\sqrt{3}}{2}\cdot x^2 + 3x\left(\frac{1000}{\sqrt{3}x^2}\right)$
$= \frac{\sqrt{3}}{2}x^2 + \frac{1000\sqrt{3}}{x}$
$= \frac{\sqrt{3}}{2}\left(x^2 + \frac{2000}{x}\right)$ ✓

(c) $A = \frac{\sqrt{3}}{2}x^2 + \frac{1000\sqrt{3}}{x}$

$A = \frac{\sqrt{3}}{2}x^2 + 1000\sqrt{3}x^{-1}$

$\frac{dA}{dx} = \sqrt{3}x + -1000\sqrt{3}x^{-2}$

$\sqrt{3}x - \frac{1000\sqrt{3}}{x^2} = 0$

$\sqrt{3}x = \frac{1000\sqrt{3}}{x^2}$

$x^3 = 1000$

$x = 10$

(d) Put $x = 10$ into A

$A = \frac{\sqrt{3}}{2}(10)^2 + \frac{1000\sqrt{3}}{10}$

$A = 50\sqrt{3} + 100\sqrt{3}$

$A = 150\sqrt{3}$

(e) Justify means find

$\frac{d^2A}{dx^2} = \sqrt{3} + 2000\sqrt{3}x^{-3}$

$\sqrt{3} + \frac{2000\sqrt{3}}{x^3}$

Put in $x = 10$ $\frac{d^2A}{dx^2} = +ive$

$$f(x) \equiv x^3 + 4x^2 + kx + 1.$$

a Find the set of values of the constant k for which the curve $y = f(x)$ has two stationary points.

Given that $k = -3$,

b find the coordinates of the stationary points of the curve $y = f(x)$.

Stationary points when $\frac{dy}{dx} = 0$ or $f'(x) = 0$

$f'(x) = 3x^2 + 8x + k$

so when $f'(x) = 0$ then $3x^2 + 8x + k = 0$

because it is a QUADRATIC

it has 2 points when

discriminent > 0

$a = 3 \quad b = 8 \quad k = c$

$b^2 - 4ac > 0$

$64 - 4(3)k > 0$

$64 > 12k$

$5\frac{1}{3} > k$

Ans $k < 5\frac{1}{3}$

b) $k = -3$

$f(x) = x^3 + 4x^2 - 3k + 1$

$f'(x) = 3x^2 + 8x - 3$

Stationary points when $f'(x) = 0$

$3x^2 + 8x - 3 = 0$

$(3x - 1)(x + 3) = 0$

$x = \frac{1}{3} \qquad x = -3$

Must find co-ordinates.

$f(\frac{1}{3}) = (\frac{1}{3})^3 + 4(\frac{1}{3})^2 - 3(\frac{1}{3}) + 1$

$= \frac{1}{27} + \frac{4}{9} - 1 + 1$

$= \frac{13}{27}$ Ans $(\frac{1}{3}, \frac{13}{27})$

$f(-3) = (-3)^3 + 4(-3)^2 - 3(-3) + 1$

$= -27 + 36 + 9 + 1$

$= 19$

Ans $(-3, 19)$

Given that $x = 2^{t-1}$ and $y = 2^{3t}$,

a find expressions in terms of t for

i xy **ii** $2y^2$

b Hence, or otherwise, find the value of t for which

$$2y^2 - xy = 0.$$

$(2^{t-1})(2^{3t})$

$= 2^{t-1+3t}$

$= 2^{4t-1}$

$2y^2$

$2(2^{3t})^2$

$2(2^{6t})$

2^{6t+1}

$2y^2 - xy = 0$

$2^{6t+1} - 2^{4t-1} = 0$

$2^{6t+1} = 2^{4t-1}$

$6t+1 = 4t-1$

$2t = -2$

$t = -1$

Should check this

$x = 2^{-1-1}$ $y = 2^{3(-1)}$

$x = 2^{-2}$ $y = 2^{-3}$

$x = \frac{1}{4}$ $y = \frac{1}{8}$

$2(\frac{1}{8})^2 - \frac{1}{4}(\frac{1}{8})$

$2(\frac{1}{64}) - \frac{1}{32} = 0$ ✓ correct

a Express $\sqrt[3]{24}$ in the form $k\sqrt[3]{3}$.

b Find the integer n such that

$$\sqrt[3]{24} + \sqrt[3]{81} = \sqrt[3]{n}.$$

Clue for $\sqrt[3]{3}$

$$\sqrt[3]{24} = (24)^{1/3} = (8 \times 3)^{1/3}$$
$$= 8^{1/3} \times 3^{1/3}$$
$$= 2 \times 3^{1/3}$$
$$= 2\sqrt[3]{3}$$

$$\sqrt[3]{81} = 81^{1/3}$$
$$= (3 \times 27)^{1/3}$$
$$= 3\sqrt[3]{3}$$

$$\sqrt[3]{24} + \sqrt[3]{81} = \sqrt[3]{n}$$
$$2\sqrt[3]{3} + 3\sqrt[3]{3} = 5\sqrt[3]{3}$$
$$= \sqrt[3]{125} \times \sqrt[3]{3}$$
$$= \sqrt[3]{375}$$

Check this!

Given

$$\frac{9^{x-1}}{3^{y+2}} = 81$$

express y in terms of x, writing your answer in simplest form.

3, 9 and 81 are all connected with 3

$$9^{x-1} = 81 \times 3^{y+2}$$
$$(3^2)^{x-1} = 3^4 \times 3^{y+2}$$
$$3^{2x-2} = 3^{y+6}$$

then

$$2x - 2 = y + 6$$
$$2x - 8 = y$$

Ans $y = 2x - 8$

A simple model for the cost of a car journey £C when a car is driven at a steady speed of v mph is

$$C = \frac{4500}{v} + v + 10$$

(a) Use this model to find the value of v which minimises the cost of the journey. **(5)**

(b) Use $\frac{d^2C}{dv^2}$ to verify that C is a minimum for this value of v **(2)**

(c) Calculate the minimum cost of the journey **(2)**

$$C = 4500v^{-1} + v + 10$$

$$\frac{dC}{dv} = -4500v^{-2} + 1$$

$$= 1 - \frac{4500}{v^2}$$

Put $\frac{dC}{dv} = 0$ to get v number

$$1 - \frac{4500}{v^2} = 0$$

$$1 = \frac{4500}{v^2}$$

$$v^2 = 4500$$

$$v = \sqrt[2]{4500} \qquad v = 30\sqrt{5}$$

Find $\frac{d^2C}{dv^2} = 9000v^{-3} = \frac{9000}{v^3}$

Put in $v = 30\sqrt{5}$

$\frac{d^2C}{dv^2} = \frac{9000}{(30\sqrt{5})^2}$ = Positive so it is MINIMUM

Minimum Cost when $v = 30\sqrt{5}$

$$C = \frac{4500}{30\sqrt{5}} + 30\sqrt{5} + 10$$

$$= \frac{150}{\sqrt{5}} + 30\sqrt{5} + 10$$

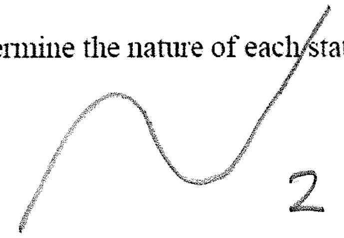
$$= 30\sqrt{5} + 30\sqrt{5} + 10$$

Ans Minimum Cost $= 10 + 60\sqrt{5}$

A curve has the equation $y = 4x^3 + 15x^2 - 18x + 5$

Find the coordinates of the stationary points and determine the nature of each stationary point.

$y = 4x^3 + 15x^2 - 18x + 5$



x^3 type
2 turning points

$\frac{dy}{dx} = 12x^2 + 30x - 18$

Stationary points are when $\frac{dy}{dx} = 0$

so solve $\frac{dy}{dx} = 0$

$12x^2 + 30x - 18 = 0$

$6(2x^2 + 5x - 3) = 0$

$6(2x - 1)(x + 3) = 0$

check FOIL or solve on calculator

$2x - 1 = 0$ $\quad$ $x + 3 = 0$

$x = 1/2$ $\quad$ $x = -3$

$y =$ $\quad$ $y =$

Determine the nature

$\frac{d^2y}{dx^2}$ must be found

if $\frac{d^2y}{dx^2} > 0$ MINIMUM t.p.

if $\frac{d^2y}{dx^2} < 0$ MAXIMUM t.p.

$\frac{d^2y}{dx^2} = 24x + 30$

Put in $x = 1/2$

$\frac{d^2y}{dx^2} > 0$

so MINIMUM at $(1/2, \quad)$

Put in $x = -3$

$\frac{d^2y}{dx^2} < 0$

so MAXIMUM at $(-3, \quad)$

Given that $6^{y+1} = 36^{x-2}$,

a express y in the form $ax + b$,

b find the value of $4^{x - \frac{1}{2}y}$.

6 and 36 are connected together by indices

$$6^{y+1} = (6^2)^{x-2}$$

$$6^{y+1} = 6^{2x-4}$$

$$y+1 = 2x-4$$

$$y = 2x-5$$

Find value of

$$4^{x - \frac{1}{2}y}$$

$$4^{x - \frac{1}{2}(2x-5)}$$

$$4^{x - x + 5/2}$$ Lucky they cancel out

$$4^{5/2}$$

$$(4^{1/2})^5$$

$$= 2^5$$

$$= 32$$

Solve the equation

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

4 and $\sqrt{2}$ are connected by the number 2

$$(2^2)^{3x-2} = 2^{-1}\,2^{-1/2}$$

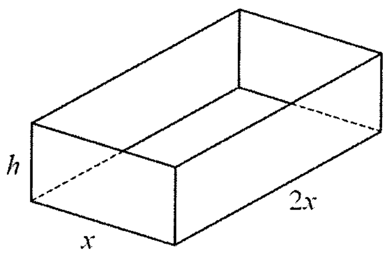
$$2^{6x-4} = 2^{-3/2}$$

$$6x - 4 = -3/2$$

$$6x = 5/2$$

$$x = 5/12$$

Remember to <u>check</u> on the calculator



Volume $= x(2x)h$

$4000 = 2x^2h$

$\frac{4000}{2x^2} = h$

The diagram shows a baking tin in the shape of an open-topped cuboid made from thin metal sheet. The base of the tin measures x cm by $2x$ cm, the height of the tin is h cm and the volume of the tin is $\boxed{4000 \text{ cm}^3}$.

a Find an expression for h in terms of x.

b Show that the area of metal sheet used to make the tin, A cm², is given by

$$A = 2x^2 + \frac{12\,000}{x}.$$

c Use differentiation to find the value of x for which A is a minimum.

d Find the minimum value of A.

e Show that your value of A is a minimum.

Area $=$ 2ends + 2sides + bottom

$= 2hx + 2(2xh) + 2x^2$

$= 6hx + 2x^2$

$= 6\left(\frac{4000}{2x^2}\right)x + 2x^2$

Area $= \frac{12000}{x} + 2x^2$

$A=$Area $= 12000x^{-1} + 2x^2$

$\frac{dA}{dx} = -12000x^{-2} + 4x$

$= \frac{-12000}{x^2} + 4x$

Put $\frac{dA}{dx} = 0$ $\quad 4x = \frac{12000}{x^2}$

$x^3 = 3000$

$x = 10\sqrt[3]{3}$

Find minimum value of A

$A = \frac{12000}{10\sqrt[3]{3}} + 2(10\sqrt[3]{3})^2$

$A =$

Show it is a minimum

Find $\frac{d^2A}{dx^2}$ and

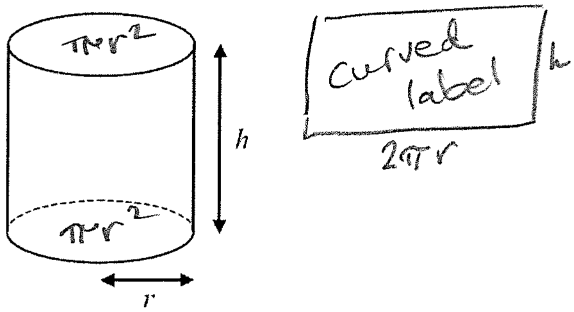
if $\frac{d^2A}{dx^2} > 0$ it is a minimum

$\frac{d^2A}{dx^2} = 24000x^{-3} + 4$

$= \frac{24000}{x^3} + 4$

Put in $x = 10\sqrt[3]{3}$

$\frac{d^2A}{dx^2} > 0$ so it is a minimum



The diagram shows a closed plastic cylinder used for making compost. The radius of the base and the height of the cylinder are r cm and h cm respectively and the surface area of the cylinder is 30 000 cm^2.

$30000 cm^2$

a Show that the volume of the cylinder, V cm^3, is given by

$$V = 15\,000r - \pi r^3.$$

b Find the maximum volume of the cylinder and show that your value is a maximum.

$$\text{Surface Area} = \underset{\text{end}}{\pi r^2} + \underset{\text{end}}{\pi r^2} + \underset{\text{wrap around}}{2\pi r h}$$

$$30000 = 2\pi r^2 + 2\pi r h$$

$$30000 - 2\pi r^2 = 2\pi r h$$

$$15000 - \pi r^2 = \pi r h$$

$$\text{Volume} = \pi r^2 \times h = r(\pi r h) = (15000 - \pi r^2) r$$

$$\text{Volume} = 15000r - \pi r^3$$

$$V = 15000r - \pi r^3$$

$$\frac{dV}{dr} = 15000 - 3\pi r^2 \quad \text{Put } \frac{dV}{dr} = 0 \text{ to get turning points}$$

$$15000 = 3\pi r^2$$

$$\frac{5000}{\pi} = r^2$$

$$\sqrt{\frac{5000}{\pi}} = r$$

Find volume
Put $r = \sqrt{\frac{5000}{\pi}}$
into $Vol = 15000r - \pi r^3$

$$\frac{d^2V}{dr^2} = -6\pi r$$

Substitute $r = \sqrt{\frac{5000}{\pi}}$ and get $\frac{d^2V}{dr^2} < 0$ which shows it is a maximum.