Given that $\int_1^4 (3x^2 + ax - 5)\ dx = 18$, find the value of the constant a.

$$\int_1^4 (3x^2 + ax - 5)\,dx$$

$$= \left[\frac{3x^3}{3} + \frac{ax^2}{2} - 5x\right]_1^4$$

$$= \left[x^3 + \frac{ax^2}{2} - 5x\right]^4 - \left[x^3 + \frac{ax^2}{2} - 5x\right]_1$$

$$= [64 + 8a - 20] - [1 + \tfrac{1}{2}a - 5]$$

$$= [44 + 8a] - [\tfrac{1}{2}a - 4]$$

But $\quad 48 + 7\tfrac{1}{2}a = 18$

$7\tfrac{1}{2}a = -30$

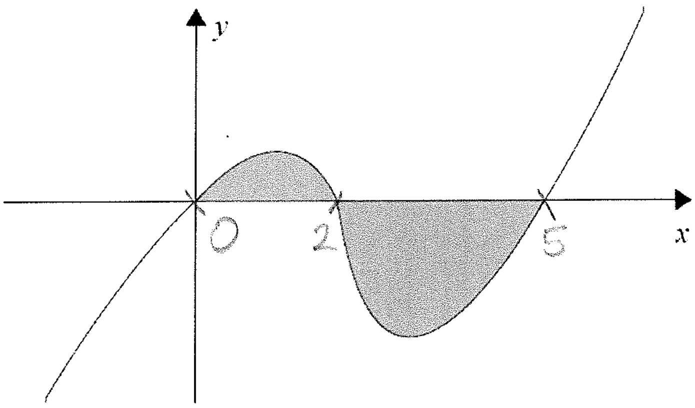
$a = -4$

Check with your new fancy white calculator

$$\int_1^4 (3x^2 - 4x - 5)\,dx = 18$$

correct

Ans $a = -4$



The sketch shows the curve $y = x(x-2)(x-5)$

(a) Write down the values of x where the curve crosses the x axis. **(1)**

(b) Find the area of the shaded region. **(8)**

Ans $x = 0$, $x = 2$ and $x = 5$

Shaded Region above x axis $= \int_0^2$ curve

" " below x axis $= \int_2^5$ curve

$y = x(x-2)(x-5)$
$y = x(x^2 - 7x + 10)$
$y = x^3 - 7x^2 + 10x$

$$\int_0^2 x^3 - 7x^2 + 10x = \left[\frac{x^4}{4} - \frac{7x^3}{3} + \frac{10x^2}{2}\right]_0^2$$

$= \ldots$

$= \frac{16}{3}$

$$\int_2^5 x^3 - 7x^2 + 10x = \left[\frac{x^4}{4} - \frac{7x^3}{3} + 5x^2\right]_2^5$$

$= \ldots$

$= -\frac{63}{4}$

Areas $= \frac{16}{3} + \frac{63}{4}$ Ans. $\frac{253}{12}$

Given that $\int_{-1}^{k} (3x^2 - 12x + 9)\ dx = 16$, find the value of the non-zero constant k.

$$\int_{-1}^{k}(3x^2 - 12x + 9)dx$$

$$\left[x^3 - 6x^2 + 9x\right]_{-1}^{k}$$

$$\left[x^3 - 6x^2 + 9x\right]^{k} - \left[x^3 - 6x^2 + 9x\right]_{-1}$$

$$\left[k^3 - 6k^2 + 9k\right] - \left[-1 - 6 - 9\right]$$

$$k^3 - 6k^2 + 9k + 16 = 16$$

so
$$k^3 - 6k^2 + 9k = 0$$

solve
$$k(k-3)(k-3) = 0$$

can't be $= 0$

$k = 3$

Check again!

$$\int_{-1}^{3}(3x^2 - 12x + 9)dx$$

Find $\int_1^3 (x+4)(x-3)\ \mathrm{d}x$

You can't integrate ( )( ) brackets like this
so use FOIL to multiply out

$(x+4)(x-3)$

$x^2 + x - 12$

$$\int_1^3 (x+4)(x-3)\,dx = \int_1^3 (x^2+x-12)\,dx$$

$$= \left[\frac{x^3}{3} + \frac{x^2}{2} - 12x\right]_1^3$$

$$= \left[\frac{x^3}{3} + \frac{x^2}{2} - 12x\right]^3 - \left[\frac{x^3}{3} + \frac{x^2}{2} - 12x\right]_1$$

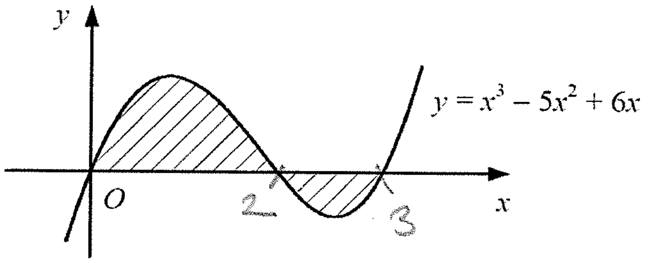
$$= \left[\frac{27}{3} + \frac{9}{2} - 36\right] - \left[\frac{1}{3} + \frac{1}{2} - 12\right]$$

$$= \left[-\frac{45}{2}\right] - \left[-\frac{67}{6}\right]$$

$$= \frac{-45}{2} + \frac{67}{6}$$

$$= -\frac{34}{3}$$

Check it using $\boxed{\int_{\square}^{\square} \square}$ button



The diagram shows the curve with the equation $y = x^3 - 5x^2 + 6x$.

a Find the coordinates of the points where the curve crosses the x-axis.

b Show that the total area of the shaded regions enclosed by the curve and the x-axis is $3\frac{1}{12}$.

$x^3 - 5x^2 + 6x = 0$

$x(x^2 - 5x + 6) = 0$

$x(x-3)(x-2) = 0$

$x = 0 \quad x = 3 \quad x = 2$

Shaded Area in 2 parts

$$\int_0^2 (x^3 - 5x^2 + 6x)\,dx = \left[\frac{x^4}{4} - \frac{5x^3}{3} + 3x^2\right]_0^2 = 8/3$$

$$\int_2^3 (x^3 - 5x^2 + 6x)\,dx = \left[\frac{x^4}{4} - \frac{5x^3}{3} + 3x^2\right]_2^3 = -5/12$$

Total Area $= 8/3 + 5/12$

$= \frac{32}{12} + \frac{5}{12}$

$= \frac{37}{12}$

$= 3\frac{1}{12}$ ✓

$f'(x) = 6x^2 - 3x + 8 \quad = \frac{dy}{dx}$

Given that the point (1, 8) lies on $y = f(x)$

Find an expression for $f(x)$

$f'(x) = 6x^2 - 3x + 8$

$f(x) = \int f'(x)\,dx = \int (6x^2 - 3x + 8)\,dx$

$f(x) = 2x^3 - \frac{3x^2}{2} + 8x + c$

BGT point (1,8) will help to get c

$8 = 2 - \frac{3}{2} + 8 + c$

$8 - 8 - 2 + 3/2 = c$

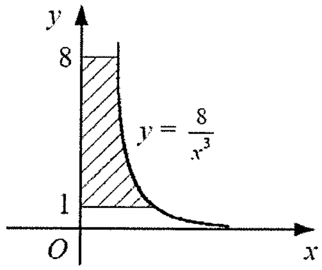
$-1/2 = c$

$f'(x) = 6x^2 - 3x + 8$

gives

$f(x) = 2x^3 - \frac{3x^2}{2} + 8x - 1/2$

a Evaluate $\int_1^2 \frac{8}{x^3}\ dx$.



$y = \frac{8}{x^3}$

$x^3 = \frac{8}{y}$

$x = \frac{2}{\sqrt[3]{y}}$

$x = 2y^{-1/3}$

The diagram shows the curve with the equation $y = \frac{8}{x^3}$, $x > 0$.

b Using your answer to part **a**, find the area of the shaded region bounded by the curve, the lines $y = 1$ and $y = 8$ and the y-axis.

(a) $\int_1^2 \frac{8}{x^3}\,dx = \int_1^2 8x^{-3}\,dx = \left[\frac{8x^{-2}}{-2}\right]_1^2$

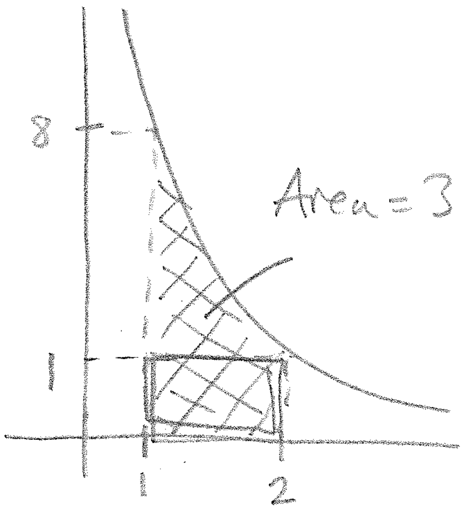
$= \left[-4x^{-2}\right]^2 - \left[-4x^{-2}\right]_1$

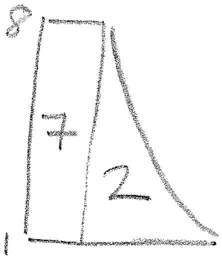
$= \left[-\frac{4}{4}\right] - \left[-\frac{4}{1}\right]$

$= -1 + 4$

$= 3$

Check it!

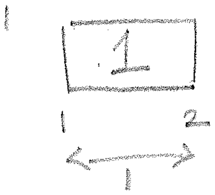




Total = 9

or

$$\int_{y=1}^{y=8} x\,dy = \int_{y=1}^{y=8} 2y^{-1/3}\,dy$$



The curve C with equation $y = f(x)$ is such that

$$\frac{dy}{dx} = 3x^2 - 4x - 1.$$

Given that the tangent to the curve at the point P with x-coordinate 2 passes through the origin, find an equation for the curve. **(7)**

$\frac{dy}{dx} = 3x^2 - 4x - 1$

$y = \int \frac{dy}{dx}\, dx = \int (3x^2 - 4x - 1)\, dx$

$\boxed{y = x^3 - 2x^2 - x + c}$

At point when $x = 2$

Gradient $m = \frac{dy}{dx} = 3\times4 - 8 - 1 = 3$

$y = 3x + c$

Passes through the origin

so $y = 3x$

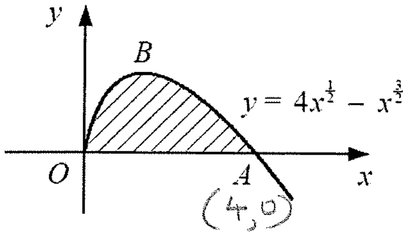
when $\boxed{x = 2 \quad y = 6}$ on straight line

so $6 = 2^3 - 2\times2^2 - 2 + c$

$6 = 8 - 8 - 2 + c$

$8 = c$

Ans $y = x^3 - 2x^2 - x + 8$ ✓



The diagram shows the curve with the equation $y = 4x^{\frac{1}{2}} - x^{\frac{3}{2}}$.

The curve meets the x-axis at the origin, O, and at the point A.

a Find the coordinates of the point A. **(2)**

c Find the area of the shaded region enclosed by the curve and the x-axis. **(4)**

$$y = 4x^{1/2} - x^{3/2}$$

when $y = 0$ $\quad 4x^{1/2} - x^{3/2} = 0$

$$x^{1/2}\left[4 - x\right] = 0$$

$$x^{1/2} = 0 \qquad x = 4$$

$$x = 0$$

$$\int_0^4 (4x^{1/2} - x^{3/2})\,dx$$

$$= \left[\frac{4x^{3/2}}{(3/2)} - \frac{x^{5/2}}{(5/2)}\right]_0^4$$

$$= \left[\frac{8x^{3/2}}{3} - \frac{2x^{5/2}}{5}\right]^4 - \left[\frac{8x^{3/2}}{3} - \frac{2x^{5/2}}{5}\right]_0$$

$$= \left[\frac{8(4)^{3/2}}{3} - \frac{2(4)^{5/2}}{5}\right] - \left[0\right]$$

$$= \left[\frac{64}{3} - \frac{64}{5}\right] - \left[0\right]$$

$$= \frac{128}{15} = 8\tfrac{8}{15}$$

Given that 3.31

$$\int_1^k \left(3 - \frac{4}{x^2}\right)\,dx = 6,$$

and that $k > 1$, find the value of the constant k.

$$\int_1^K (3 - 4x^{-2})\,dx$$

$$\left[3x - \frac{4x^{-1}}{-1}\right]_1^k$$

$$= \left[3x + \frac{4}{x}\right]_1^k$$

$$= \left[3x + \frac{4}{x}\right]^k - \left[3x + \frac{4}{x}\right]_1$$

$$= \left[3k + \frac{4}{k}\right] - \left[3 + 4\right]$$

$$= 3k + \frac{4}{k} - 7$$

$$= 6$$

so $$3k + \frac{4}{k} - 7 = 6$$

$$3k + \frac{4}{k} = 13$$

$$3k + \frac{4}{k} = 13$$

$$3k^2 + 4 = 13k$$

$$3k^2 - 13k + 4 = 0$$

$$(3k-1)(k-4) = 0$$

$k = 1/3$ $\quad$ $k = 4$

Ans $k = 4$
Check it!

Find $\int (x+4)(x-3)\ dx$

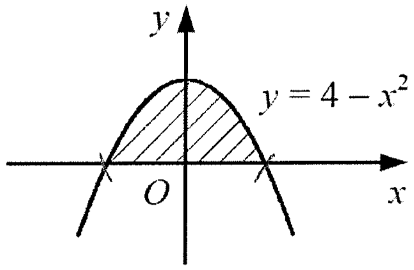
$\int (x+4)(x-3)\,dx$

You can integrate (X) easily

Do FOIL

$\int x^2 + x - 12\ dx$

$= x^3/3 + x^2/2 - 12x + c$



The diagram shows the curve with the equation $y = 4 - x^2$.

a Find the coordinates of the points where the curve crosses the x-axis.

b Find the area of the shaded region enclosed by the curve and the x-axis.

$y = 4 - x^2$

crosses x axis when $y = 0$

$0 = 4 - x^2$

$0 = (2 - x)(2 + x)$ D.O.T.S.

$x = 2 \qquad x = -2$

$$\int_{-2}^{2} (4 - x^2)\,dx = \left[4x - \frac{x^3}{3}\right]_{-2}^{2}$$

$$= \left[4x - \frac{x^3}{3}\right]^{2} - \left[4x - \frac{x^3}{3}\right]_{-2}$$

$$= \left[8 - \frac{8}{3}\right] - \left[-8 + \frac{8}{3}\right]$$

$$= \left[\frac{16}{3}\right] - \left[-\frac{16}{3}\right]$$

$$= \frac{32}{3}$$

$$= 10\tfrac{2}{3}$$