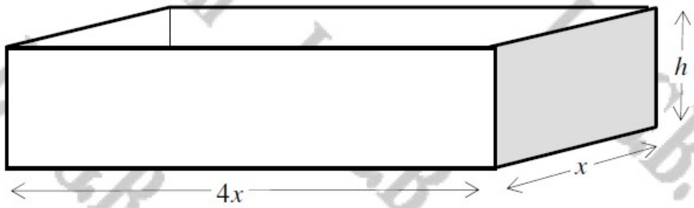


The figure above shows a box in the shape of a cuboid with a rectangular base x cm by $4x$ cm and **no top**. The height of the box is h cm.

It is given that the surface area of the box is 1728 cm^2.

a) Show clearly that

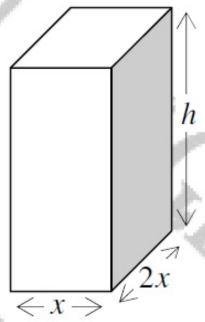
$$h = \frac{864 - 2x^2}{5x}.$$

b) Use part **(a)** to show that the volume of the box, $V \text{ cm}^3$, is given by

$$V = \tfrac{8}{5}\left(432x - x^3\right).$$

c) Find the value of x for which V is stationary.

d) Find the maximum value for V, fully justifying the fact that it is the maximum.



The figure above shows the design of a fruit juice carton with capacity of 1000 cm^3.

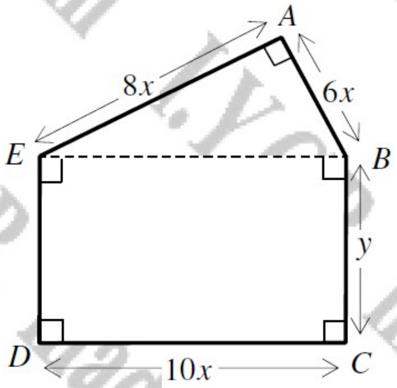
The design of the carton is that of a closed cuboid whose base measures x cm by $2x$ cm, and its height is h cm.

a) Show that the surface area of the carton, $A \text{ cm}^2$, is given by

$$A = 4x^2 + \frac{3000}{x}.$$

b) Find the value of x for which A is stationary.

c) Calculate the minimum value for A, justifying fully the fact that it is indeed the minimum value of A.



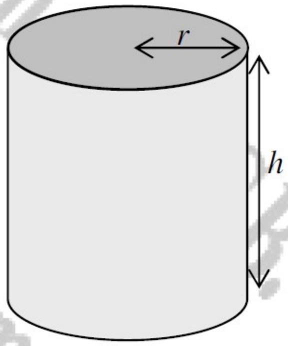
The figure above shows a pentagon $ABCDE$ whose measurements, in cm, are given in terms of x and y.

a) If the perimeter of the pentagon is 120 cm, show clearly that its area, A cm^2, is given by

$$A = 600x - 96x^2.$$

b) Use a method based on differentiation to calculate the maximum value for A, fully justifying the fact that it is indeed the maximum value.

I.Y.G.B. madasmaths.com



The figure above shows a **closed** cylindrical can of radius r cm and height h cm.

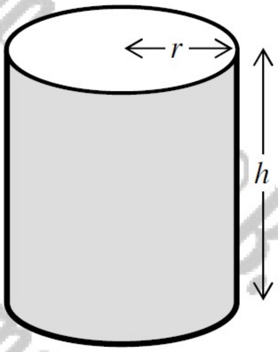
a) Given that the surface area of the can is 192π cm^2, show that the volume of the can, V cm^3, is given by

$$V = 96\pi r - \pi r^3.$$

b) Find the value of r for which V is stationary.

c) Justify that the value of r found in part **(b)** gives the maximum value for V.

d) Calculate the maximum value of V.



A pencil holder is in the shape of a right circular cylinder, which is **open** at one of its circular ends.

The cylinder has radius r cm and height h cm and the total surface area of the cylinder, including its base, is 360 cm^2.

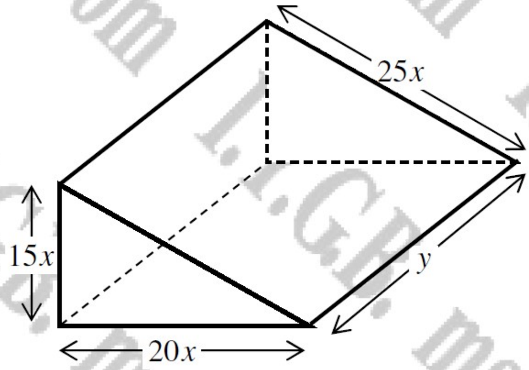
a) Show that the volume, $V \text{ cm}^3$, of the cylinder is given by

$$V = 180r - \frac{1}{2}\pi r^3 .$$

b) Determine by differentiation the value of r for which V has a stationary value.

c) Show that the value of r found in part (b) gives the maximum value for V .

d) Calculate, to the nearest cm^3, the maximum volume of the pencil holder.



The figure above shows a solid triangular prism with a **total** surface area of 3600 cm^2.

The triangular faces of the prism are right angled with a base of $20x$ cm and a height of $15x$ cm. The length of the prism is y cm.

a) Show that the volume of the prism, V cm^3, is given by

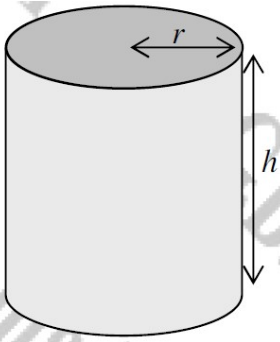
$$V = 9000x - 750x^3.$$

b) Find the value of x for which V is stationary.

c) Show that the value of x found in part (b) gives the maximum value for V.

d) Determine the value of y when V becomes maximum.

I.Y.G.B. madasmaths.com



The figure above shows a **closed** cylindrical can, of radius r cm and height h cm.

a) If the volume of the can is 330 cm^3, show that surface area of the can, $A \text{ cm}^2$, is given by

$$A = 2\pi r^2 + \frac{660}{r}.$$

b) Find the value of r for which A is stationary.

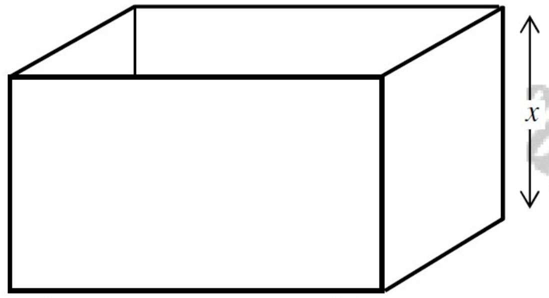
c) Justify that the value of r found in part **(b)** gives the minimum value for A.

d) Hence calculate the minimum value of A.

I.Y.G.B. madasmaths.com

The figure below shows a large tank in the shape of a cuboid with a **rectangular** base and **no top**.

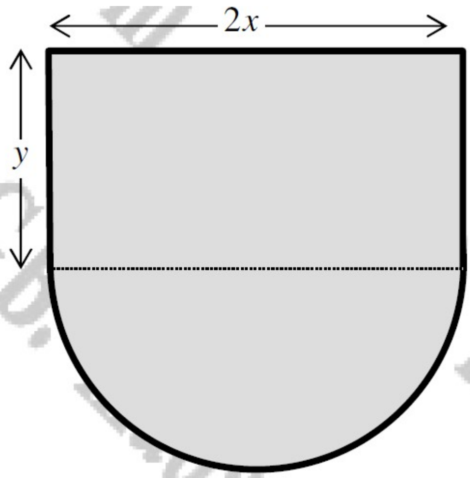
Two of the vertical opposite faces of the cuboid are square and the height of the cuboid is x metres.



a) Given that the surface area of the tank is 54 m^2, show that the capacity of the tank, $V \text{ m}^3$, is given by

$$V = 18x - \frac{2}{3}x^3 .$$

b) Find the maximum value for V, fully justifying the fact that it is indeed the maximum value.



The figure above shows the design of a theatre stage which is the shape of a semicircle attached to rectangle. The diameter of the semicircle is $2x$ m and is attached to one side of the rectangle also measuring $2x$ m. The other side of the rectangle is y m.

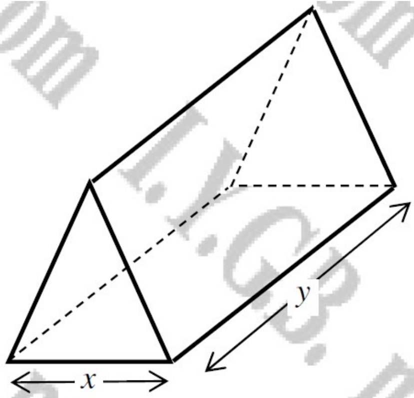
The perimeter of the stage is 60 m.

a) Show that the total area of the stage, A m^2, is given by

$$A = 60x - 2x^2 - \frac{1}{2}\pi x^2.$$

b) Show further, by using a **differentiation** method, that the maximum area of the stage is

$$\frac{1800}{\pi + 4} \text{ m}^2.$$



The figure above shows a triangular prism whose triangular faces are parallel to each other and are in the shape of **equilateral** triangles of side length x cm.

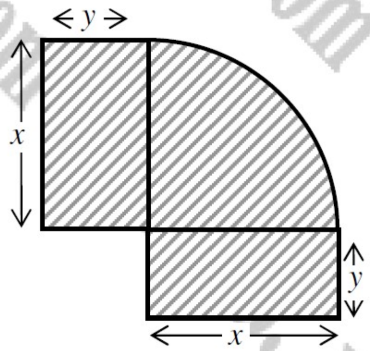
The length of the prism is y.

a) Given that total surface area of the prism is exactly $54\sqrt{3}$ cm^2, show clearly that the volume of the prism, V cm^3, is given by

$$V = \frac{27}{2}x - \frac{1}{8}x^3.$$

b) Find the maximum value of V, fully justifying the fact that it is indeed the maximum value.

c) Determine the value of y when V takes this maximum value.



The figure above shows the design for an earring consisting of a quarter circle with two identical rectangles attached to either straight edge of the quarter circle. The quarter circle has radius x cm and the each of the rectangles measure x cm by y cm.

The earring is assumed to have negligible thickness and treated as a two dimensional object with area 12.25 cm^2.

a) Show that the perimeter, P cm, of the earring is given by

$$P = 2x + \frac{49}{2x}.$$

b) Find the value of x that makes the perimeter of the earring minimum, fully justifying that this value of x produces a minimum perimeter.

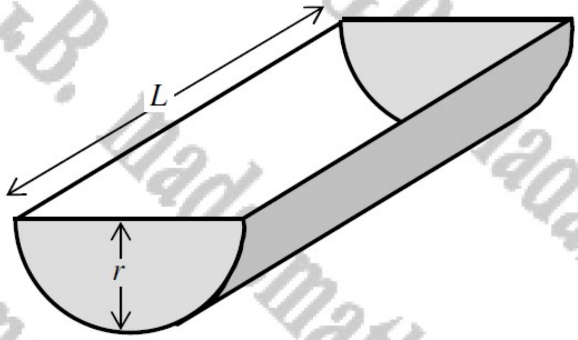
c) Show that for the value of x found in part **(b)**, the corresponding value of y is

$$\frac{7}{16}(4-\pi).$$

I.Y.G.B. madasmaths.com

The figure below shows the design of an animal feeder which in the shape of a hollow, open topped half cylinder, made of thin sheet metal. The radius of the semicircular ends is r cm and the length of the feeder is L cm.

The metal used in the construction of the feeder is $600\pi \text{ cm}^2$.



a) Show that the capacity, V cm^3, of the feeder is given by

$$V = 300\pi r - \frac{1}{2}\pi r^3.$$

The design of the feeder is such so its capacity is maximum.

b) Determine the exact value of r for which V is stationary.

c) Show that the value of r found in part (b) gives the maximum value for V.

d) Find, in exact form, the capacity and the length of the feeder.