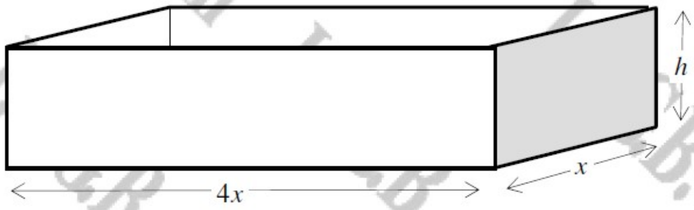


The figure above shows a box in the shape of a cuboid with a rectangular base x cm by $4x$ cm and **no top**. The height of the box is h cm.

It is given that the surface area of the box is 1728 cm^2.

a) Show clearly that

$$h = \frac{864 - 2x^2}{5x}.$$

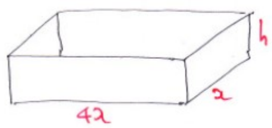
b) Use part **(a)** to show that the volume of the box, $V \text{ cm}^3$, is given by

$$V = \frac{8}{5}\left(432x - x^3\right).$$

c) Find the value of x for which V is stationary.

d) Find the maximum value for V, fully justifying the fact that it is the maximum.

(a)



$$1728 = 4x^2 + 2(xh) + 2(4xh)$$
$$1728 = 4x^2 + 2xh + 8xh$$
$$1728 = 4x^2 + 10xh$$
$$864 = 2x^2 + 5xh$$
$$864 - 2x^2 = 5xh$$
$$h = \frac{864 - 2x^2}{5x}$$

AS REQUIRED

(b)
$$V = 4x^2h$$
$$V = 4x^{\cancel{2}} \times \frac{864 - 2x^2}{5\cancel{x}}$$
$$V = \frac{4x(864 - 2x^2)}{5}$$
$$V = \frac{8x(432 - x^2)}{5}$$
$$V = \frac{8}{5}(432x - x^3)$$

AS REQUIRED

(c)
$$\frac{dV}{dx} = \frac{8}{5}\left(432 - 3x^2\right)$$

FOR MIN/MAX $\frac{dV}{dx} = 0$

$$\frac{8}{5}(432 - 3x^2) = 0$$
$$432 - 3x^2 = 0$$
$$3x^2 = 432$$
$$x^2 = 144$$
$$x = 12 \quad (x > 0)$$

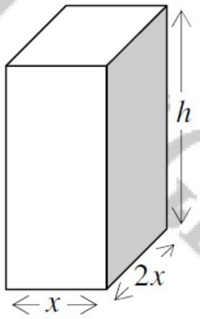
(d)
$$\frac{d^2V}{dx^2} = \frac{8}{5}(-6x) = -\frac{48}{5}x$$

$$\left.\frac{d^2V}{dx^2}\right|_{x=12} = -\frac{48 \times 12}{5} = -\frac{576}{5} < 0$$

INDEED A MAXIMUM

USING $x = 12$

$$V_{MAX} = \frac{8}{5}\left(432 \times 12 - 12^3\right)$$
$$V_{MAX} = \frac{8}{5} \times 3456$$
$$V_{MAX} = 5529.6 \text{ cm}^3$$



The figure above shows the design of a fruit juice carton with capacity of 1000 cm^3.

The design of the carton is that of a closed cuboid whose base measures x cm by $2x$ cm, and its height is h cm.

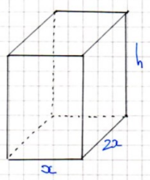
a) Show that the surface area of the carton, $A \text{ cm}^2$, is given by

$$A = 4x^2 + \frac{3000}{x}.$$

b) Find the value of x for which A is stationary.

c) Calculate the minimum value for A, justifying fully the fact that it is indeed the minimum value of A.

a)



CONSTRAINT $V = 1000 \text{ cm}^3$

$\Rightarrow V = x(2x)h$

$\Rightarrow 1000 = 2x^2h$

$\Rightarrow x^2h = 500$

$xh = \frac{500}{x}$

SURFACE AREA (A cm^2)

$\Rightarrow A = 2\left[2x^2 + xh + 2xh\right]$

$\Rightarrow A = 4x^2 + 6xh$

$\Rightarrow A = 4x^2 + \frac{3000}{x}$ $\boxed{6xh = \frac{3000}{x}}$

AS REQUIRED

b) $A = 4x^2 + 3000x^{-1}$

$\Rightarrow \frac{dA}{dx} = 8x - 3000x^{-2}$

FOR STATIONARY VALUES $\frac{dA}{dx} = 0$

$\Rightarrow 8x - \frac{3000}{x^2} = 0$

$\Rightarrow 8x = \frac{3000}{x^2}$

$\Rightarrow 8x^3 = 3000$

$\Rightarrow x^3 = 375$

$\Rightarrow x = \sqrt[3]{375} \approx 7.21 \text{ cm}$

c) $A = 4x^2 + \frac{3000}{x}$

$\Rightarrow A_{MIN} = 4(7.21...)^2 + \frac{3000}{7.21...}$

$\Rightarrow A_{MIN} \approx 624 \text{ cm}^2$

TO JUSTIFY IT IS A MIN, USE 2ND DERIVATIVE

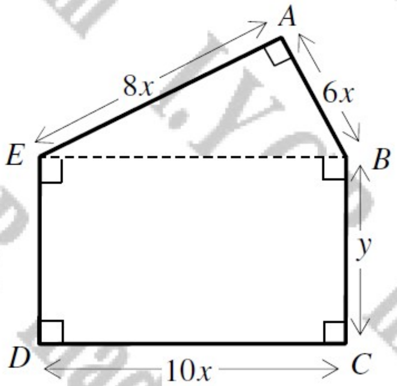
$\Rightarrow \frac{dA}{dx} = 8x - 3000x^{-2}$

$\Rightarrow \frac{d^2A}{dx^2} = 8 + 6000x^{-3}$

$\Rightarrow \frac{d^2A}{dx^2} = 8 + \frac{6000}{x^3}$

$\Rightarrow \left.\frac{d^2A}{dx^2}\right|_{x=7.21...} = 8 + \frac{6000}{(7.21...)^3} = 24 > 0$

INDEED A MINIMUM



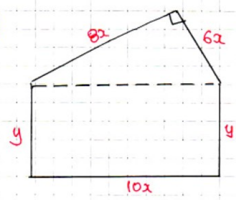
The figure above shows a pentagon $ABCDE$ whose measurements, in cm, are given in terms of x and y.

a) If the perimeter of the pentagon is 120 cm, show clearly that its area, A cm^2, is given by

$$A = 600x - 96x^2.$$

b) Use a method based on differentiation to calculate the maximum value for A, fully justifying the fact that it is indeed the maximum value.

a)



CONSTRAINT

$P = 120$

$2y + 10x + 8x + 6x = 120$

$2y + 24x = 120$

$y + 12x = 60$

$y = 60 - 12x$

"MAIN EQUATION"

AREA $= A = 10xy + \frac{1}{2}(8x)(6x)$

$A = 10xy + 24x^2$

$A = 10x(60 - 12x) + 24x^2$

$A = 600x - 120x^2 + 24x^2$

$A = 600x - 96x^2$ // AS REQUIRED

b)

DIFFERENTIATE W.R.T x & SOLVE FOR ZERO

$\frac{dA}{dx} = 600 - 192x$

$0 = 600 - 192x$

$192x = 600$

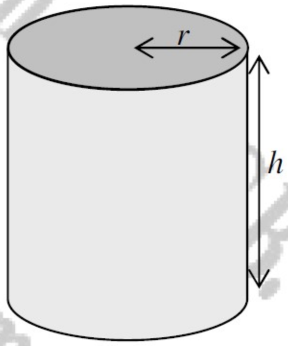
$x = \frac{25}{8} = 3.125$

$\therefore A_{MAX} = 600(3.125) - 96(3.125)^2 = 937.5$

JUSTIFYING IT IS A MAX

$\frac{d^2A}{dx^2} = -192$

$\left.\frac{d^2A}{dx^2}\right|_{x=3.125} = -192 < 0$ INDEED A MAX //



The figure above shows a **closed** cylindrical can of radius r cm and height h cm.

a) Given that the surface area of the can is $192\pi \text{ cm}^2$, show that the volume of the can, $V \text{ cm}^3$, is given by

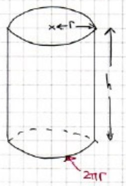
$$V = 96\pi r - \pi r^3 .$$

b) Find the value of r for which V is stationary.

c) Justify that the value of r found in part **(b)** gives the maximum value for V.

d) Calculate the maximum value of V.

a)



CONSTRAINT SURFACE AREA = 192π

$\Rightarrow h \times 2\pi r + 2\times\pi r^2 = 192\pi$

$\Rightarrow 2\cancel{\pi}rh + 2\times\cancel{\pi}r^2 = 192\cancel{\pi}$

$\Rightarrow rh + r^2 = 96$

$rh = 96 - r^2$

VOLUME = $\pi r^2 \times h$

$\Rightarrow V = \pi r(rh)$

$\Rightarrow V = \pi r(96 - r^2)$

$\Rightarrow V = 96\pi r - \pi r^3$ AS REQUIRED

b) DIFFERENTIATE & SOLVE FOR ZERO

$\Rightarrow V = 96\pi r - \pi r^3$

$\Rightarrow \frac{dV}{dr} = 96\pi - 3\pi r^2$

$\Rightarrow 0 = 96\pi - 3\pi r^2$

$\Rightarrow 3\cancel{\pi}r^2 = 96\cancel{\pi}$

$\Rightarrow r^2 = 32$

$\Rightarrow r = +\sqrt{32} \approx 5.66 \text{ cm}$

c) CHECKING WITH THE 2ND DERIVATIVE

$\Rightarrow \frac{dV}{dr} = 96\pi - 3\pi r^2$

$\Rightarrow \frac{d^2V}{dr^2} = -6\pi r$

$\Rightarrow \left.\frac{d^2V}{dr^2}\right|_{r=5.66} = -106.62\ldots < 0$

INDEED IT WILL GIVE THE MAXIMUM VALUE FOR V

d) $V = 96\pi r - \pi r^3$

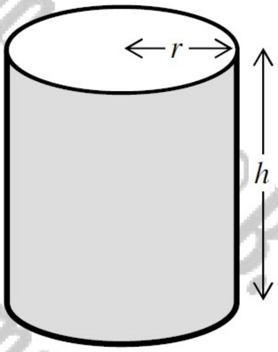
$\Rightarrow V_{MAX} = 96\pi(4\sqrt{2}) - \pi(4\sqrt{2})^3$

$\Rightarrow V_{MAX} = 4\pi\sqrt{2}\left[96 - (4\sqrt{2})^2\right]$

$\Rightarrow V_{MAX} = 4\pi\sqrt{2} \times 64$

$\Rightarrow V_{MAX} = 256\pi\sqrt{2}$

$\Rightarrow V_{MAX} \simeq 1137$



A pencil holder is in the shape of a right circular cylinder, which is **open** at one of its circular ends.

The cylinder has radius r cm and height h cm and the total surface area of the cylinder, including its base, is 360 cm^2.

a) Show that the volume, $V\text{ cm}^3$, of the cylinder is given by

$$V = 180r - \frac{1}{2}\pi r^3 .$$

b) Determine by differentiation the value of r for which V has a stationary value.

c) Show that the value of r found in part (b) gives the maximum value for V.

d) Calculate, to the nearest cm^3, the maximum volume of the pencil holder.

(a)

SURFACE AREA = 360

$\pi r^2 + 2\pi r h = 360$

$2\pi r h = 360 - \pi r^2$

$\boxed{h = \frac{360 - \pi r^2}{2\pi r}}$

$V = \pi r^2 h$

$V = \cancel{\pi} r^{\cancel{2}} \times \frac{360 - \pi r^2}{2\cancel{\pi}\cancel{r}}$

$V = \frac{360r - \pi r^3}{2}$

$V = 180r - \frac{1}{2}\pi r^3$ AS REQUIRED

(b) $\frac{dV}{dr} = 180 - \frac{3}{2}\pi r^2$

SOLVING FOR ZERO

$\Rightarrow 180 - \frac{3}{2}\pi r^2 = 0$

$\Rightarrow 180 = \frac{3}{2}\pi r^2$

$\Rightarrow 360 = 3\pi r^2$

$\Rightarrow \frac{120}{\pi} = r^2$

$\Rightarrow r = \sqrt[2]{\frac{120}{\pi}} \simeq 6.18$

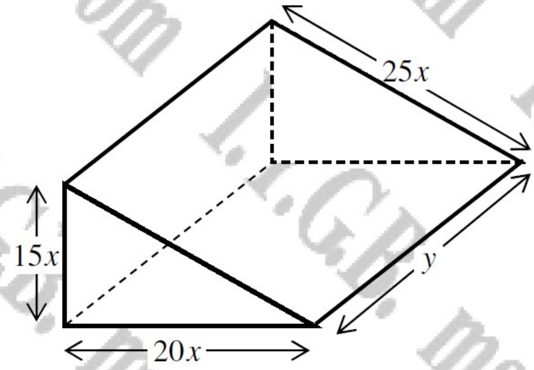
(c) $\frac{d^2V}{dr^2} = -3\pi r$

$\left.\frac{d^2V}{dr^2}\right|_{r=6.18..} = -3\pi(6.18) = -58.3.. < 0$

$\therefore$ INDEED A MAXIMUM

(d) $V_{max} = 180(6.18) - \frac{1}{2}\pi(6.18)^3$

$V_{max} = 742$ NEAREST cm^3



The figure above shows a solid triangular prism with a **total** surface area of 3600 cm^2.

The triangular faces of the prism are right angled with a base of $20x$ cm and a height of $15x$ cm. The length of the prism is y cm.

a) Show that the volume of the prism, $V \text{ cm}^3$, is given by

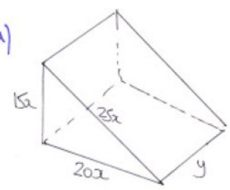
$$V = 9000x - 750x^3 .$$

b) Find the value of x for which V is stationary.

c) Show that the value of x found in part (b) gives the maximum value for V.

d) Determine the value of y when V becomes maximum.

(a)



$A = 3600$

$3600 = 15xy + 20xy + 25xy + \cancel{2} \times \cancel{\tfrac{1}{2}} \times (15x)(20x)$

$3600 = 60xy + 300x^2$

$\boxed{60 = xy + 5x^2}$

$xy = 60 - 5x^2$

$V = \frac{1}{2}(15x)(20x)y$

$V = 150x^2 y$

$V = 150x(xy)$

$V = 150x(60 - 5x^2)$

$V = 9000x - 750x^3$ // AS REQUIRED

(b) $\frac{dV}{dx} = 9000 - 2250x^2$

SOLVE FOR ZERO

$9000 - 2250x^2 = 0$

$x^2 = 4$

$x = 2$ // $(x > 0)$

(c) $\frac{d^2V}{dx^2} = -4500x$

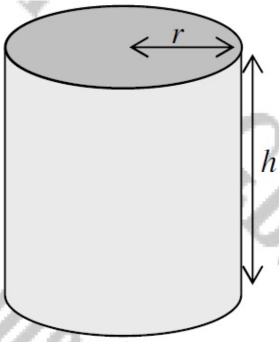
$\left.\frac{d^2V}{dx^2}\right|_{x=2} = -9000 < 0$

INDEED A MAXIMUM //

(d) USING $xy = 60 - 5x^2$

$2y = 60 - 20$

$y = 20$ //



The figure above shows a **closed** cylindrical can, of radius r cm and height h cm.

a) If the volume of the can is 330 cm^3, show that surface area of the can, $A \text{ cm}^2$, is given by

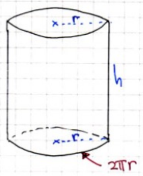
$$A = 2\pi r^2 + \frac{660}{r}.$$

b) Find the value of r for which A is stationary.

c) Justify that the value of r found in part **(b)** gives the minimum value for A.

d) Hence calculate the minimum value of A.

a)



CONSTRAINT ON THE VOLUME

$V = 330$

$\pi r^2 h = 330$

$(\pi r h) r = 330$

$\pi r h = \frac{330}{r}$

$2\pi r h = \frac{660}{r}$

$A = \pi r^2 \times 2 + (2\pi r \times h)$

$A = 2\pi r^2 + 2\pi r h$

$A = 2\pi r^2 + \frac{660}{r}$

AS REQUIRED

b) DIFFERENTIATE & SOLVE FOR ZERO

$A = 2\pi r^2 + 660 r^{-1}$

$\frac{dA}{dr} = 4\pi r - 660 r^{-2}$

$\frac{dA}{dr} = 4\pi r - \frac{660}{r^2}$

FOR MIN/MAX $\frac{dA}{dr} = 0$

$0 = 4\pi r - \frac{660}{r^2}$

$\frac{660}{r^2} = 4\pi r$

$660 = 4\pi r^3$

$r^3 = \frac{165}{\pi}$

$r = 3.745 \text{ cm}$

c) USING THE SECOND DERIVATIVE

$\frac{dA}{dr} = 4\pi r - 660 r^{-2}$

$\frac{d^2A}{dr^2} = 4\pi + 1320 r^{-3}$

$\left.\frac{d^2A}{dr^2}\right|_{r=3.745\ldots} = 12\pi \approx 37.7 > 0$

INDEED $r = 3.745$ MINIMIZES A

d) FINALLY USING

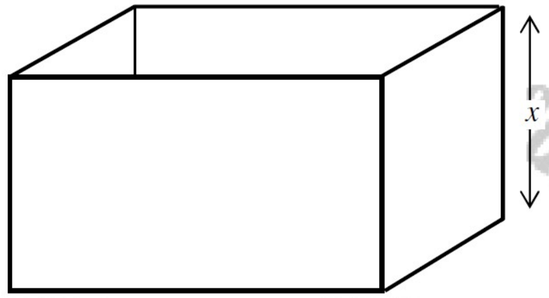
$A = 2\pi r^2 + \frac{660}{r}$

$A_{MIN} = 2\pi (3.745\ldots)^2 + \frac{660}{3.745}$

$A_{MIN} \approx 264 \text{ cm}^2$

The figure below shows a large tank in the shape of a cuboid with a **rectangular** base and **no top**.

Two of the vertical opposite faces of the cuboid are square and the height of the cuboid is x metres.

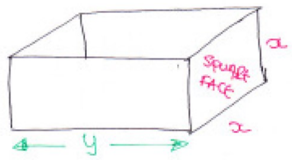


a) Given that the surface area of the tank is 54 m^2, show that the capacity of the tank, $V \text{ m}^3$, is given by

$$V = 18x - \frac{2}{3}x^3.$$

b) Find the maximum value for V, fully justifying the fact that it is indeed the maximum value.

(a)



• LET LENGTH BE y

$2x^2 + 2xy + xy = 54$

$2x^2 + 3xy = 54$

$3xy = 54 - 2x^2$

$\boxed{xy = 18 - \frac{2}{3}x^2}$

$V = x^2 y$

$V = x(xy)$

$V = x\left(18 - \frac{2}{3}x^2\right)$

$V = 18x - \frac{2}{3}x^3$

AS REQUIRED

(b) • $\frac{dV}{dx} = 18 - 2x^2$

SOLVE FOR ZERO

$18 - 2x^2 = 0$

$2x^2 = 18$

$x^2 = 9$

$x = 3 \quad (x > 0)$

• $\frac{d^2V}{dx^2} = -4x$

$\left.\frac{d^2V}{dx^2}\right|_{x=3} = -12 < 0$

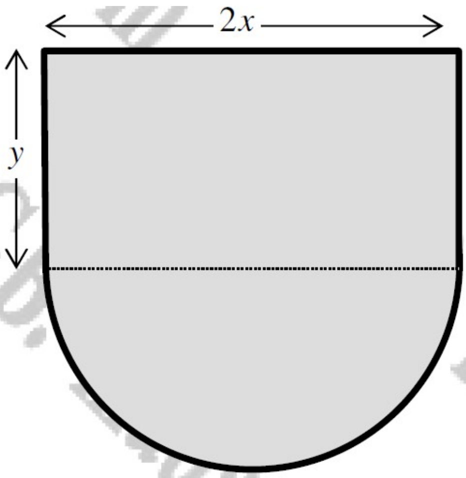
INDEED A MAXIMUM

$V_{MAX} = 18 \times 3 - \frac{2}{3} \times 3^3$

$V_{MAX} = 54 - \frac{2}{3} \times 27$

$V_{MAX} = 54 - 18$

$V_{MAX} = 36$



The figure above shows the design of a theatre stage which is the shape of a semicircle attached to rectangle. The diameter of the semicircle is $2x$ m and is attached to one side of the rectangle also measuring $2x$ m. The other side of the rectangle is y m.

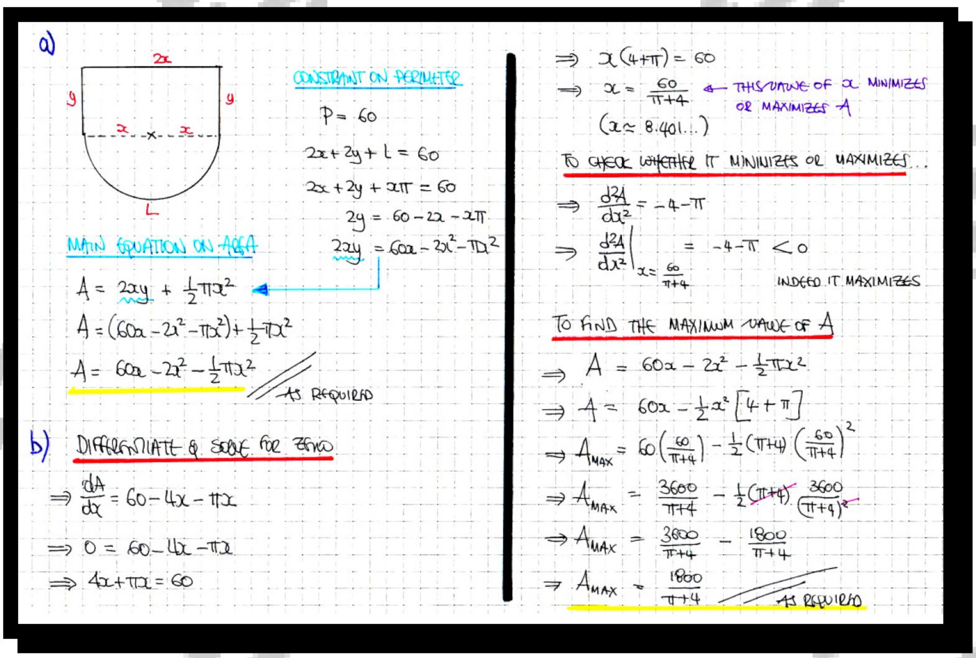
The perimeter of the stage is 60 m.

a) Show that the total area of the stage, A m^2, is given by

$$A = 60x - 2x^2 - \frac{1}{2}\pi x^2 .$$

b) Show further, by using a **differentiation** method, that the maximum area of the stage is

$$\frac{1800}{\pi + 4}\ \text{m}^2 .$$



a)

$2x$, y, y, x, x, L

CONSTRAINT ON PERIMETER

$P = 60$

$2x + 2y + L = 60$

$2x + 2y + x\pi = 60$

$2y = 60 - 2x - x\pi$

$2xy = 60x - 2x^2 - \pi x^2$

MAIN EQUATION ON AREA

$A = 2xy + \frac{1}{2}\pi x^2$

$A = (60x - 2x^2 - \pi x^2) + \frac{1}{2}\pi x^2$

$A = 60x - 2x^2 - \frac{1}{2}\pi x^2$ // AS REQUIRED

b) DIFFERENTIATE & SOLVE FOR ZERO

$\Rightarrow \frac{dA}{dx} = 60 - 4x - \pi x$

$\Rightarrow 0 = 60 - 4x - \pi x$

$\Rightarrow 4x + \pi x = 60$

$\Rightarrow x(4 + \pi) = 60$

$\Rightarrow x = \frac{60}{\pi + 4}$ ← THIS VALUE OF x MINIMIZES OR MAXIMIZES A

$(x \simeq 8.401\ldots)$

TO CHECK WHETHER IT MINIMIZES OR MAXIMIZES…

$\Rightarrow \frac{d^2A}{dx^2} = -4 - \pi$

$\Rightarrow \left.\frac{d^2A}{dx^2}\right|_{x = \frac{60}{\pi + 4}} = -4 - \pi < 0$

INDEED IT MAXIMIZES

TO FIND THE MAXIMUM VALUE OF A

$\Rightarrow A = 60x - 2x^2 - \frac{1}{2}\pi x^2$

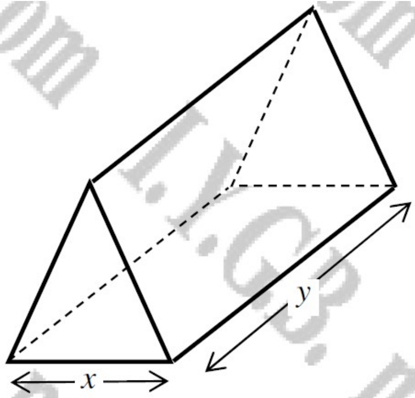
$\Rightarrow A = 60x - \frac{1}{2}x^2 [4 + \pi]$

$\Rightarrow A_{MAX} = 60\left(\frac{60}{\pi + 4}\right) - \frac{1}{2}(\pi + 4)\left(\frac{60}{\pi + 4}\right)^2$

$\Rightarrow A_{MAX} = \frac{3600}{\pi + 4} - \frac{1}{2}\cancel{(\pi + 4)}\frac{3600}{(\pi + 4)^{\cancel{2}}}$

$\Rightarrow A_{MAX} = \frac{3600}{\pi + 4} - \frac{1800}{\pi + 4}$

$\Rightarrow A_{MAX} = \frac{1800}{\pi + 4}$ // AS REQUIRED



The figure above shows a triangular prism whose triangular faces are parallel to each other and are in the shape of **equilateral** triangles of side length x cm.

The length of the prism is y.

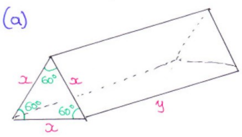
a) Given that total surface area of the prism is exactly $54\sqrt{3}$ cm^2, show clearly that the volume of the prism, V cm^3, is given by

$$V = \frac{27}{2}x - \frac{1}{8}x^3.$$

b) Find the maximum value of V, fully justifying the fact that it is indeed the maximum value.

c) Determine the value of y when V takes this maximum value.

(a)



$S = 54\sqrt{3}$

$2\left(\frac{1}{2}x^2\sin 60°\right) + 3(xy) = 54\sqrt{3}$

$x^2\frac{\sqrt{3}}{2} + 3xy = 54\sqrt{3}$

$\boxed{x^2\sqrt{3} + 6xy = 108\sqrt{3}}$

$6xy = 108\sqrt{3} - x^2\sqrt{3}$

$\boxed{xy = 18\sqrt{3} - \frac{1}{6}x^2\sqrt{3}}$

$V = \left(\frac{1}{2}x^2\sin 60°\right)y$

$V = \frac{1}{2}x^2\frac{\sqrt{3}}{2}y$

$V = \frac{1}{4}\sqrt{3}x^2y$

$V = \frac{1}{4}\sqrt{3}x(xy)$

$V = \frac{1}{4}\sqrt{3}x\left(18\sqrt{3} - \frac{1}{6}x^2\sqrt{3}\right)$

$V = \frac{27}{2}x - \frac{1}{8}x^3$

AS REQUIRED

(b) • $\frac{dV}{dx} = \frac{27}{2} - \frac{3}{8}x^2$

SOLVE FOR ZERO

$\frac{27}{2} - \frac{3}{8}x^2 = 0$

$\frac{27}{2} = \frac{3}{8}x^2$

$x^2 = 36$

$x = 6$ $(x > 0)$

• $\frac{d^2V}{dx^2} = -\frac{3}{4}x$

$\left.\frac{d^2V}{dx^2}\right|_{x=6} = -\frac{9}{2} < 0$

INDEED A MAXIMUM

• WHEN $x = 6$

$V = \frac{27}{4}\times 6 - \frac{1}{16}\times 6^3$

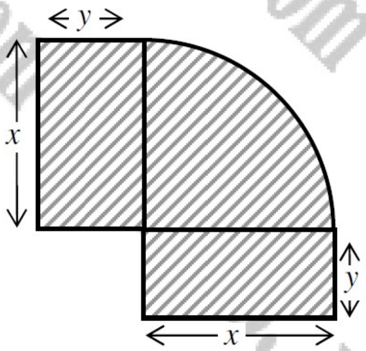
$V = 27$

(c) NOW $xy = 18\sqrt{3} - \frac{1}{6}x^2\sqrt{3}$

$6y = 18\sqrt{3} - 6\sqrt{3}$

$6y = 12\sqrt{3}$

$y = 2\sqrt{3}$



The figure above shows the design for an earring consisting of a quarter circle with two identical rectangles attached to either straight edge of the quarter circle. The quarter circle has radius x cm and the each of the rectangles measure x cm by y cm.

The earring is assumed to have negligible thickness and treated as a two dimensional object with area 12.25 cm^2.

a) Show that the perimeter, P cm, of the earring is given by

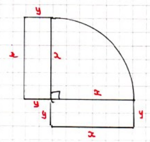
$$P = 2x + \frac{49}{2x}.$$

b) Find the value of x that makes the perimeter of the earring minimum, fully justifying that this value of x produces a minimum perimeter.

c) Show that for the value of x found in part **(b)**, the corresponding value of y is

$$\frac{7}{16}(4-\pi).$$

a)



CONSTRAINT ON AREA

$A = 12.25$

$2xy + \frac{1}{4} \times \pi x^2 = 12.25$ $\times 4$

$8xy + \pi x^2 = 49$

$\frac{8xy}{2x} + \frac{\pi x^2}{2x} = \frac{49}{2x}$ $\div 2x$

$4y + \frac{\pi}{2}x = \frac{49}{2x}$

$4y = \frac{49}{2x} - \frac{1}{2}\pi x$

PERIMETER $= 2x + 4y + \frac{1}{4}(2\pi x)$

$\Rightarrow P = 2x + 4y + \frac{1}{2}\pi x$

$\Rightarrow P = 2x + \left(\frac{49}{2x} - \cancel{\frac{1}{2}\pi x}\right) + \cancel{\frac{1}{2}\pi x}$

$\Rightarrow P = 2x + \frac{49}{2x}$ // AS REQUIRED

b) DIFFERENTIATE & SOLVE FOR ZERO

$\Rightarrow P = 2x + \frac{49}{2}x^{-1}$

$\Rightarrow \frac{dP}{dx} = 2 - \frac{49}{2}x^{-2}$

FOR MIN/MAX $\frac{dP}{dx} = 0$

$\Rightarrow 2 - \frac{49}{2x^2} = 0$

$\Rightarrow 2 = \frac{49}{2x^2}$

$\Rightarrow 4x^2 = 49$

$\Rightarrow x^2 = 12.25$

$\Rightarrow x = 3.5 \quad (x > 0)$

USE 2ND DERIVATIVE TO JUSTIFY MINIMUM.

$\Rightarrow \frac{dP}{dx} = 2 - \frac{49}{2}x^{-2}$

$\Rightarrow \frac{d^2P}{dx^2} = 49x^{-3} = \frac{49}{x^3}$

$\Rightarrow \left.\frac{d^2P}{dx^2}\right|_{x=3.5} = \frac{8}{7} > 0$

INDEED $x = 3.5$ MINIMIZES P

c) USING THE CONSTRAINT EQUATION

$\Rightarrow 8xy + \pi x^2 = 49$

$\Rightarrow 8(3.5)y + \pi(3.5)^2 = 49$

$\Rightarrow 28y + \frac{49}{4}\pi = 49$ $\div 7$

$\Rightarrow 4y + \frac{7}{4}\pi = 7$ $\times 4$

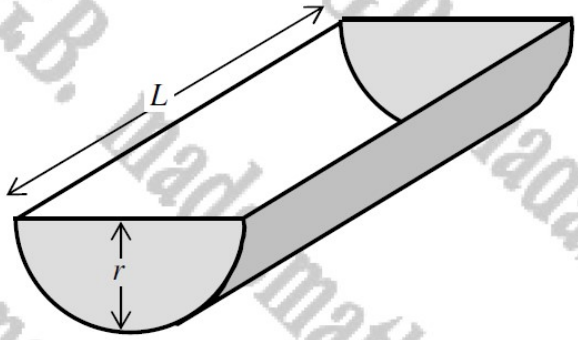
$\Rightarrow 16y + 7\pi = 28$

$\Rightarrow 16y = 28 - 7\pi$

$\Rightarrow y = \frac{7(4-\pi)}{16}$ // AS REQUIRED

The figure below shows the design of an animal feeder which in the shape of a hollow, open topped half cylinder, made of thin sheet metal. The radius of the semicircular ends is r cm and the length of the feeder is L cm.

The metal used in the construction of the feeder is $600\pi \text{ cm}^2$.



a) Show that the capacity, V cm^3, of the feeder is given by

$$V = 300\pi r - \frac{1}{2}\pi r^3.$$

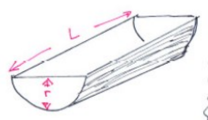
The design of the feeder is such so its capacity is maximum.

b) Determine the exact value of r for which V is stationary.

c) Show that the value of r found in part (b) gives the maximum value for V.

d) Find, in exact form, the capacity and the length of the feeder.

a)



$V = \frac{1}{2}(\pi r^2 L)$

$\Rightarrow V = \frac{1}{2}\pi r^2 \times \frac{600 - r^2}{r}$

$\Rightarrow V = \frac{\pi r^2(600 - r^2)}{2r}$

$\Rightarrow V = \frac{\pi r(600 - r^2)}{2}$

$\Rightarrow V = \frac{600\pi r}{2} - \frac{\pi r^3}{2}$

$\Rightarrow V = 300\pi r - \frac{1}{2}\pi r^3$ // AS REQUIRED

$\Rightarrow A = 600\pi$

$\Rightarrow \pi r^2 + \frac{1}{2}(2\pi r L) = 600\pi$

2 SEMICIRCLES CURVED SECTION

$\Rightarrow \pi r^2 + \pi r L = 600\pi$

$\Rightarrow r^2 + rL = 600$

$\Rightarrow rL = 600 - r^2$

$\Rightarrow \boxed{L = \frac{600 - r^2}{r}}$

b) $\frac{dV}{dr} = 300\pi - \frac{3}{2}\pi r^2$

SOLVE FOR ZERO TO GET $\Rightarrow$ $300\pi - \frac{3}{2}\pi r^2 = 0$

$600 - 3r^2 = 0$

$600 = 3r^2$

$r^2 = 200$

$r = 10\sqrt{2}$ // $(r > 0)$

c) $\frac{d^2V}{dr^2} = -3\pi r$

$\left.\frac{d^2V}{dr^2}\right|_{r = 10\sqrt{2}} = -30\pi\sqrt{2} < 0$ INDEED A MAXIMUM //

d) • $V = 300\pi(10\sqrt{2}) - \frac{1}{2}\pi(10\sqrt{2})^3$

$V = 3000\pi\sqrt{2} - \frac{1}{2}\pi(1000 \times 2\sqrt{2})$

$V = 3000\pi\sqrt{2} - 1000\pi\sqrt{2}$

$V = 2000\pi\sqrt{2}$ //

& • $L = \frac{600 - (10\sqrt{2})^2}{10\sqrt{2}}$

$L = \frac{600 - 200}{10\sqrt{2}}$

$L = \frac{40}{\sqrt{2}}$

$L = 20\sqrt{2}$ //