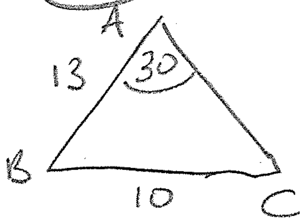
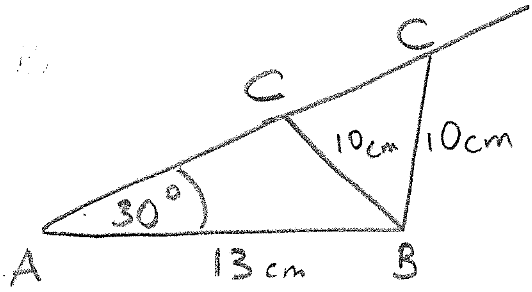
In the triangle ABC, $AB = 13$cm, $BC = 10$cm and angle $BAC = 30°$

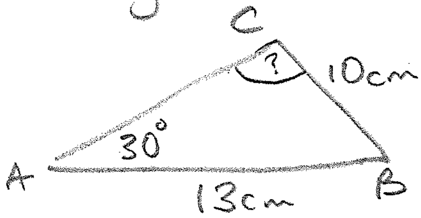
Find the two possible sizes of angle ABC, giving your answers to two decimal places.



Now draw more carefully.



This is the ambiguous case.



sine Rule

$$\frac{\sin C}{13} = \frac{\sin 30}{10}$$

$$\sin C = \frac{13 \sin 30}{10}$$

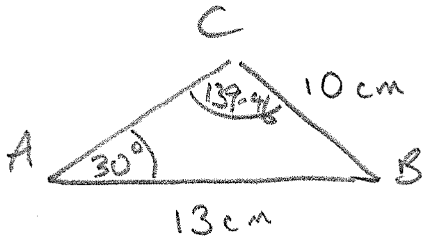
$$\sin C = \frac{13}{20}$$

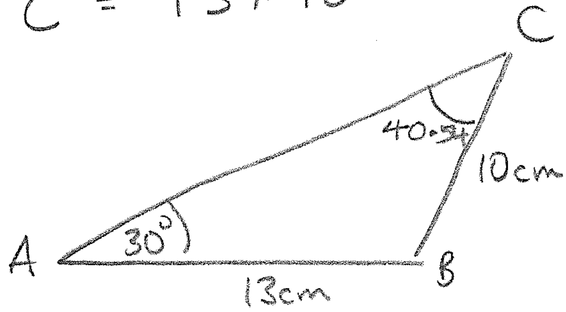
$$C = \sin^{-1}\left(\frac{13}{20}\right)$$

$$C = 40.54°$$

But also

$$C = 139.46°$$



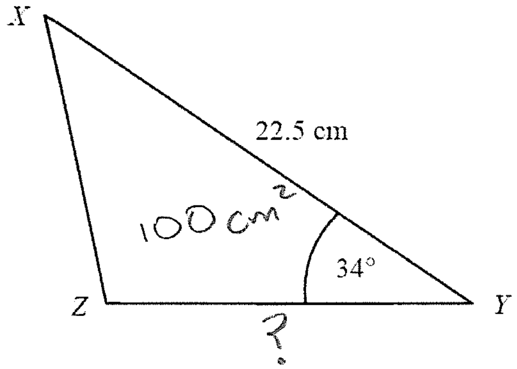


Ans

$ABC = 10.54°$

Ans $ABC = 109.46°$

Check it



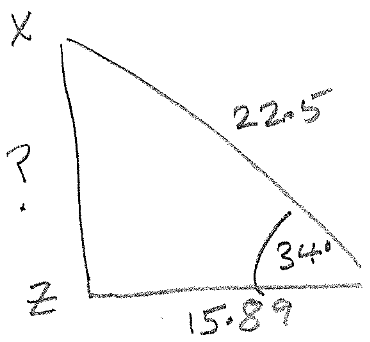
The diagram shows triangle XYZ in which $XY = 22.5$ cm and $\angle XYZ = 34°$.
Given that the area of the triangle is $100\ \text{cm}^2$, find the length XZ.

"Area $= \frac{1}{2} ab\sin C$"

Find ZY first

$100 = \frac{1}{2} \times 22.5 \times ? \times \sin 34$

$15.89 = ?$



Use Cosine Rule because you know 3 sides and 1 angle

$XZ^2 = 22.5^2 + 15.89^2 - 2 \times 22.5 \times 15.89 \cos 34$

$XZ^2 = 166.13$

$XZ = \sqrt{166.13}$

$XZ = 12.89$

Ans 12.9

ALWAYS GIVE FINAL ANSWER TO 3 SIG. FIG.
BUT use more accurate in your working

$$f(x) = x^3 + 6x^2 + px + q$$

Given that f(4) = 0 and f(-5) = 36

(a) Find the values of p and q

simultaneous equations will be important

(b) Factorise f(x) completely.

$f(4) = 4^3 + 6(4)^2 + 4p + q = 0$

$160 + 4p + q = 0$

$f(-5) = (-5)^3 + 6(-5)^2 + (-5)p + q = 36$

$25 - 5p + q = 36$

$\boxed{160 + 4p + q = 0}$ $\boxed{q - 5p = 11}$

solve simultaneous eq

$160 + 4p + 11 + 5p = 0$

$9p = -171$

$p = -19$

$q = -84$

Ans $p = -19$ and $q = -84$

Factorise

$x^3 + 6x^2 - 19x - 84$

$(x-4)$ is a factor

because of Factor Theorem

long division

$$\begin{array}{r|l} & x^2 + 10x + 21 \\ \hline x - 4 & x^3 + 6x^2 - 19x - 84 \\ & \cancel{x^3} - 4x^2 \\ \hline & 10x^2 - 19x \\ & \cancel{10x^2} - 40x \\ \hline & 21x - 84 \end{array}$$

$x^2 + 10x + 21$

$(x+7)(x+3)$

You can check with EQUATION SOLVER

Ans $(x-4)(x+7)(x+3)$

Perhaps long division instead

$$f(x) = 2x^3 - 13x^2 + 8x + 48$$

(a) Prove that $(x - 4)$ is a factor of f(x). FACTOR THEOREM

$f(4) = 2\times(4)^3 - 13\times 4^2 + 8(4) + 48$

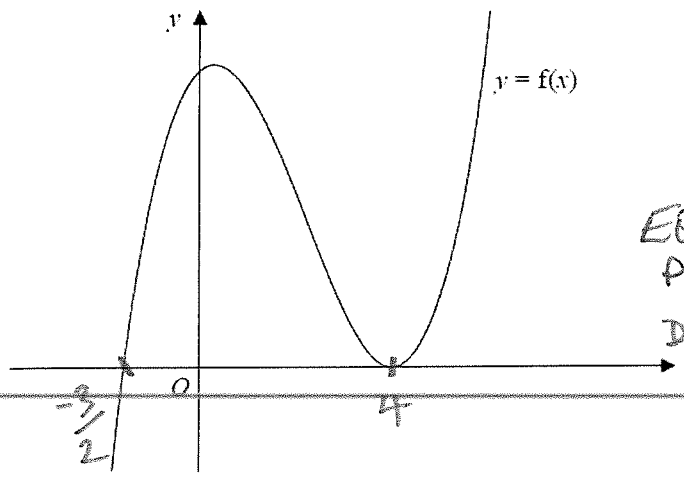
$= 128 - 208 + 32 + 48$

$= 0$ so $(x-4)$ is factor.

(2)

(b) Hence, using algebra, show that the equation f(x) = 0 has only two distinct roots.

(4)



Just use fancy calculator

EQUATION SOLVER

POLYNOMIAL

DEGREE 3

$x = -\frac{3}{2}$

$x = 4$

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 - 13x^2 + 8x + 46 = 0$$

(2)

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(d) find the two possible values of k.

(2)

(b) Long division

$$\begin{array}{r} 2x^2 - 5x - 12 \\ x-4 \,\big)\, 2x^3 - 13x^2 + 8x + 48 \\ 2x^3 - 8x^2 \qquad\qquad \\ -5x^2 + 8x \qquad \\ -5x^2 + 20x \qquad \\ -12x + 48 \end{array}$$

$(x-4)(2x^2 - 5x - 12)$

$(x-4)^2(2x-3)$

one solution other solution

(c) Using sketch $x = 4$ repeated root

(d) $f(x+k)$ moves k units left

TRANSFORMATION

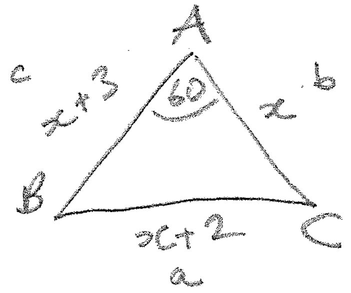
$f(x+4)$ and $f(x - \frac{3}{2})$

Beyoncé "to the left"

Ans $k = 4$ & $-\frac{3}{2}$

In the triangle ABC, $AB = (x + 3)$cm, $BC = (x + 2)$ cm, $AC = x$ cm and angle $BAC = 60°$

Find the value of x.



Looks like Cosine Rule because you know 3 sides & 1 angle

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$(x+2)^2 = x^2 + (x+3)^2 - 2(x)(x+3)\cos 60°$$

$$(x+2)^2 = x^2 + (x+3)^2 - x(x+3)$$

$$x^2+4x+4 = x^2 + x^2+6x+9 - x^2 - 3x$$

$$0 = \cancel{2x^2} - \cancel{2x^2} + 6x - 3x - 4x + 9 - 4$$

$$0 = 5 - x$$

Ans $x = 5$

You should check

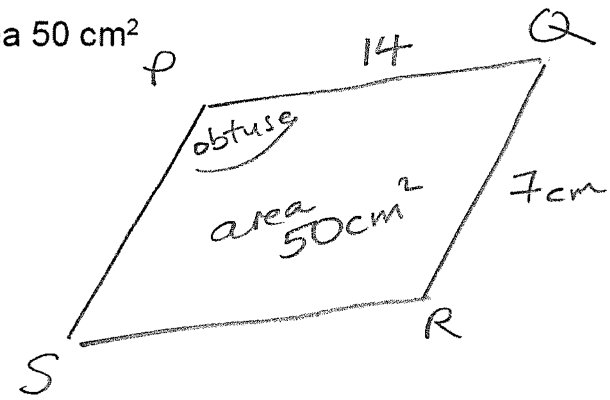
Put $x=5$ back into this

$$7^2 = 5^2 + 8^2 - 2(5)(8)\cos 60°$$

✓

Answer $x = 5$

A parallelogram *PQRS* has area 50 cm²



Given

- *PQ* has length 14 cm
- *QR* has length 7 cm
- angle *SPQ* is obtuse

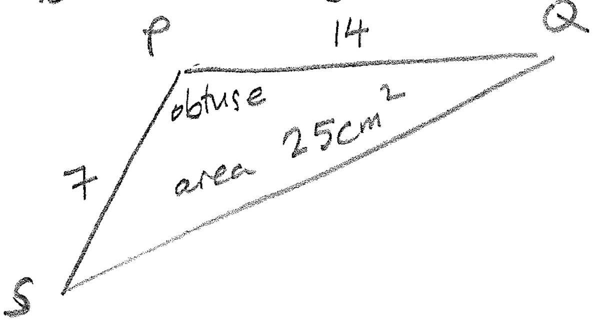
find

(a) the size of angle *SPQ*, in degrees, to 2 decimal places,

(b) the length of the diagonal *SQ*, in cm, to one decimal place.

---

look at only

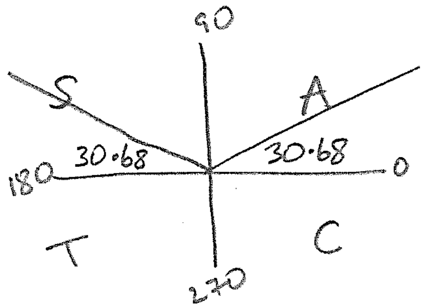


"Area $= \frac{1}{2} ab\sin C$"

$$25 = \frac{1}{2} \times 7 \times 14 \times \sin \hat{P}$$

$$\frac{50}{98} = \sin \hat{P}$$

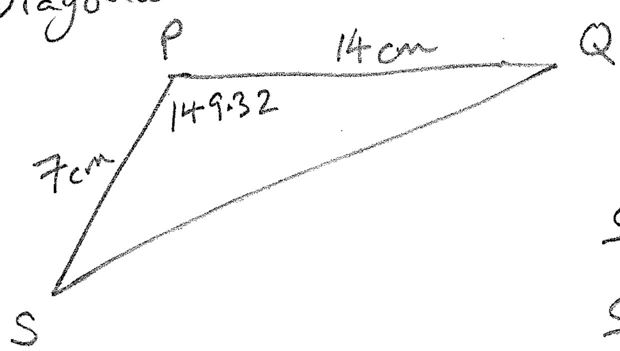
$$\frac{25}{49} = \sin \hat{P}$$



Angle $S\hat{P}Q$ is obtuse

$$S\hat{P}Q = 149.32^\circ$$

Diagonal



Cosine Rule

$$SQ^2 = 7^2 + 14^2 - 2 \times 7 \times 14 \times \cos 149.32$$

$$SQ^2 = 413.57$$

$$SQ = 20.34$$

Ans $SQ = 20.3$ cm

$$f(x) \equiv px^3 + qx^2 + qx + 3.$$

Given that $(x + 1)$ is a factor of $f(x)$,

a find the value of the constant p.

Given also that when $f(x)$ is divided by $(x - 2)$ the remainder is 15,

b find the value of the constant q.

$(x+1)$ is a factor so $f(-1) = 0$ Remainder/ Factor Theorem

$f(-1) = p(-1)^3 + q(-1)^2 + (-1)q + 3$

$= -p + q - q + 3$

$= 3 - p$ so $3 - p = f(-1) = 0$

Ans $\boxed{p = 3}$

$f(x) = 3x^3 + qx^2 + qx + 3$

$(x-2)$ has remainder 15

so $f(2) = 15$

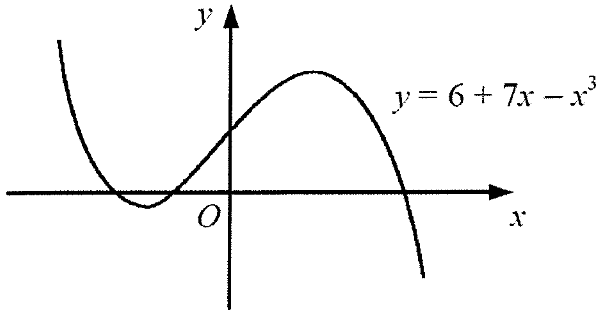
$f(2) = 24 + 4q + 2q + 3 = 15$

$6q = 15 - 27$

$6q = -12$

$q = -2$

Answer $\boxed{q = -2}$



The diagram shows the curve with the equation $y = 6 + 7x - x^3$.

Find the coordinates of the points where the curve crosses the x-axis.

REALLY EASY WITH FANCY CALCULATOR

$\boxed{\text{EQUATION SOLVER}}$ $\boxed{\text{POLYNOMIAL}}$ $\boxed{\text{DEGREE} = 3}$

gives really quickly $x = 3$ $x = -1$ $x = -2$

BUT we need method

Try Remainder/Factor Theorem

$f(1) = 6 + 7(1) - (1)^3 = 12$ $(x-1)$ not a factor

$f(-1) = 6 + 7(-1) - (-1)^3 = 6 - 7 + 1 = 0$

so $(x+1)$ is a factor

you could try others like

$(x+2)$ so $f(-2)$ you know

$(x-3)$ so $f(3)$ these work

or long division

$$6 + 7x - x^3 = -(x^3 - 7x - 6)$$
$$= -(x+1)(x+2)(x-3)$$

These gives co-ordinates

$(-1,0)$ $(-2,0)$ and $(3,0)$

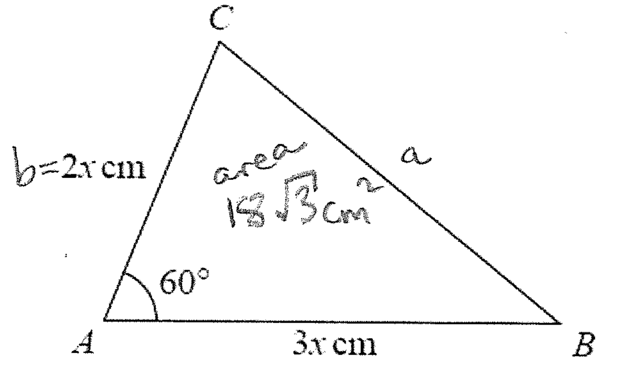


Figure 1

Figure 1 shows a sketch of a triangle ABC with $AB = 3x$ cm, $AC = 2x$ cm and angle $CAB = 60°$

Given that the area of triangle ABC is $18\sqrt{3}$ cm^2

(a) show that $x = 2\sqrt{3}$

(b) Hence find the exact length of BC, giving your answer as a simplified surd.

(a) $\text{Area} = \frac{1}{2} bc \sin A$

$18\sqrt{3} = \frac{1}{2} \times (2x) \times (3x) \sin 60$

$18\sqrt{3} = 3x^2\left(\frac{\sqrt{3}}{2}\right)$

$18 = \frac{3x^2}{2}$

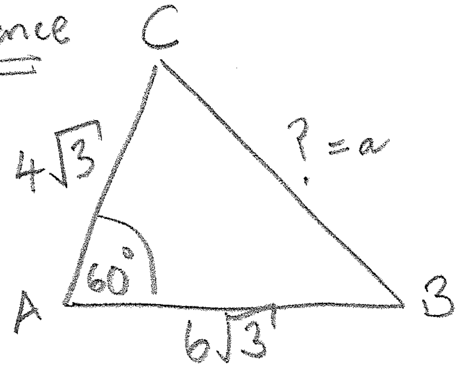
$12 = x^2$

$x = \sqrt{12}$

$x = \sqrt{4}\sqrt{3}$

$x = 2\sqrt{3}$

(b) Hence



$a^2 = b^2 + c^2 - 2bc\cos A$

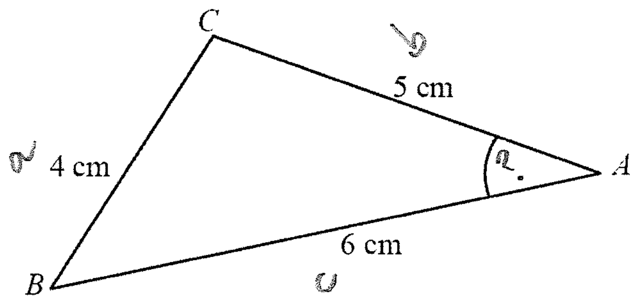
$a^2 = (4\sqrt{3})^2 + (6\sqrt{3})^2 - 2(4\sqrt{3})(6\sqrt{3})\frac{1}{2}$

$a^2 = 48 + 108 - 72$

$a^2 = 84$

$a = \sqrt{84}$

$a = 2\sqrt{21}$



The diagram above shows the triangle ABC, with $AB = 6$ cm, $BC = 4$ cm and $CA = 5$ cm.

(a) Show that $\cos A = \frac{3}{4}$.

3 sides & 1 angle means Cosine Rule

(b) Hence, or otherwise, find the exact value of $\sin A$.

$a^2 = b^2 + c^2 - 2bc \cos A$

$4^2 = 5^2 + 6^2 - 2 \times 5 \times 6 \cos A$

$16 = 25 + 36 - 60 \cos A$

$60 \cos A = 25 + 36 - 16$

$60 \cos A = 45$

$\cos A = \frac{45}{60}$

$\cos A = \frac{3}{4}$ must be acute ... think S A / T C

$\sin A = ?$

What links $\sin A$ and $\cos A$ together?

$\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$\sin^2 A = 1 - \left(\frac{3}{4}\right)^2$

$\sin^2 A = 1 - \frac{9}{16}$

$\sin^2 A = \frac{7}{16}$ $\qquad \sin A = \frac{\sqrt{7}}{4}$

$$g(x) = 4x^3 - 12x^2 - 15x + 50$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $g(x)$.

(2)

(b) Hence show that $g(x)$ can be written in the form $g(x) = (x + 2)(ax + b)^2$, where a and b are integers to be found.

(4)

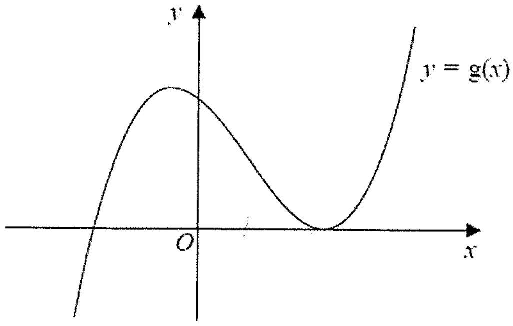


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = g(x)$

$(x+2)$ means try $g(-2)$

$g(-2) = 4(-2)^3 - 12(-2)^2 - 15(-2) + 50$

$= -32 - 48 + 30 + 50$

$g(-2) = 0$

Long Division by $(x+2)$

$$4x^2 - 20x + 25$$

$$x+2 \;\big)\; \cancel{4x^3} - 12x^2 - 15x + 50$$

$$\cancel{4x^3} + 8x^2$$

$$\cancel{-20x^2} - 15x$$

$$-20x^2 - 40x$$

$$25x + 50$$

$$25x + 50$$

$$0$$

$(x+2)(4x^2-20x+25)$

$(x+2)(2x-5)(2x-5)$

$(x+2)(2x-5)^2$

Ans $a = 2$ & $b = -5$

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant a.

Factor Theorem

$(x+2)$ is a factor means $f(-2) = 0$

$f(-2) = 2(-2)^3 - 5(-2)^2 + a(-2) + a$

$= -16 - 20 - 2a + a$

$= -36 - a$

so $-36 - a = 0$

$a = 36$

You SHOULD check on your NEW AMAZING CALCULATOR

$\boxed{\text{solve}}$ $\boxed{\text{Polynomial}}$ $\boxed{\text{Degree 3}}$