$$f(x) = (x + 3)(x + 2)(x - 1)$$

(a) Sketch the curve $y = f(x)$, showing the points of intersection with the coordinate axis. **(3)**

(b) Showing the coordinates of the points of intersection with the coordinate axis, sketch on separate diagrams the curves

(i) $y = f(x - 3)$ **(2)**

(ii) $y = f(-x)$ **(2)**

$f(x) = (x+3)(x+2)(x-1)$

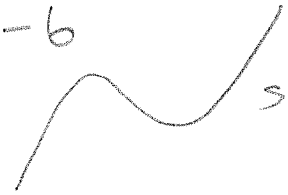
cuts x axis when $f(x) = 0$

$(x+3)(x+2)(x-1) = 0$

when $x = -3$ $x = -2$ and $x = 1$

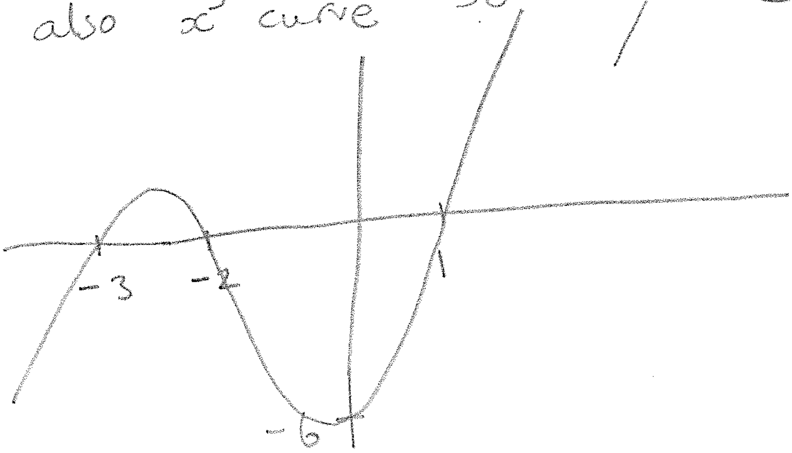
cuts y axis when $x = 0$

$f(0) = (3)(2)(-1) = -6$



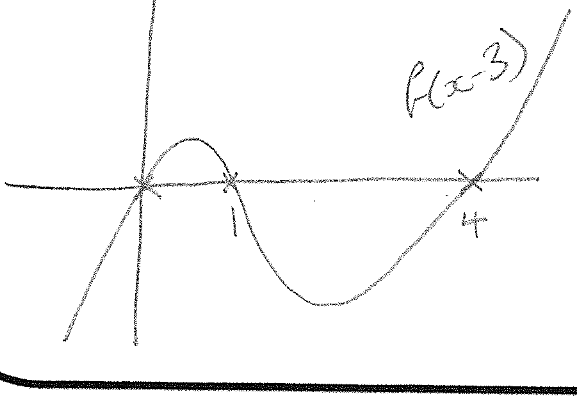
shape

also x^3 curve so



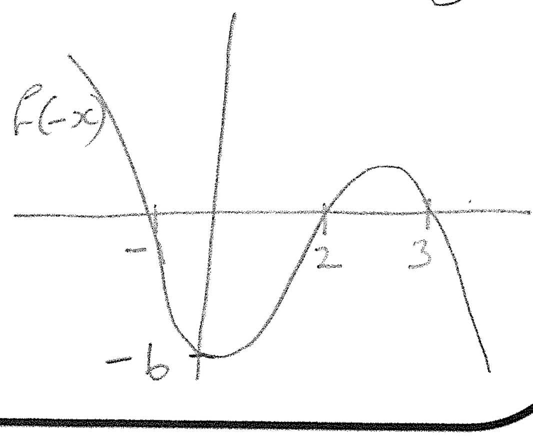
$f(x-3)$

moves 3 $\longrightarrow$



$f(-x)$ $\longleftrightarrow$

means swap over across the y axis



$$f(x) = x^3 + 4x^2 - 5x$$

(a) Sketch the curve $y = f(x)$, showing the points of intersection with the coordinate axis. **(3)**

(b) Showing the coordinates of the points of intersection with the coordinate axis, sketch on separate diagrams the curves

(i) $y = f(x + 1)$ **(2)**

(ii) $y = f(2x)$ **(2)**

$f(x) = x^3 + 4x^2 - 5x$

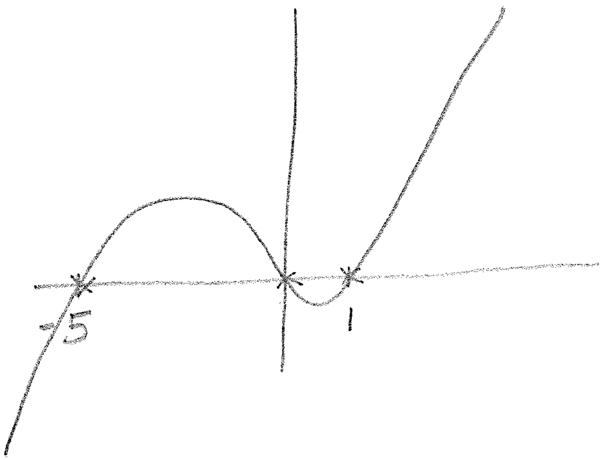
$f(x) = x(x^2 + 4x - 5)$

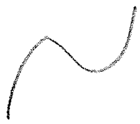
$f(x) = x(x+5)(x-1)$

cuts x axis when $x = 0$ $x = -5$ $x = 1$

where does it cut y axis

when $x = 0$ $(0,0)$



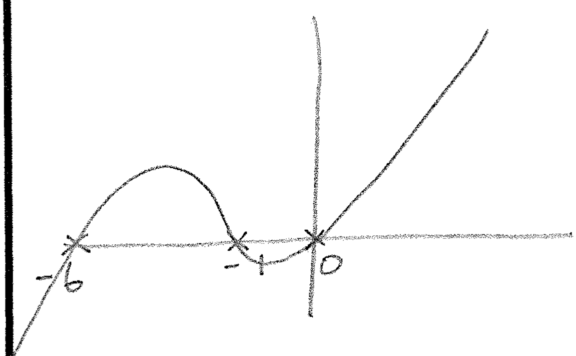


x^3 shape

$f(x+1)$

Beyoncé transformation

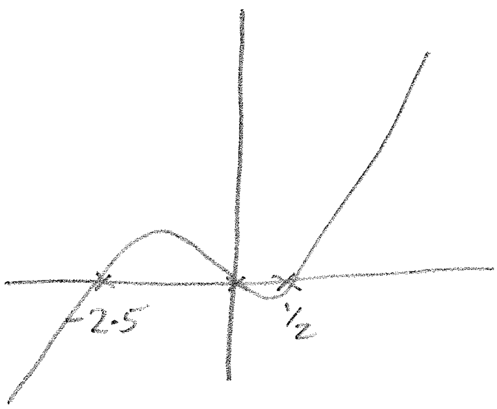
1 unit to the left



$f(2x)$

squashes in $\times 2$

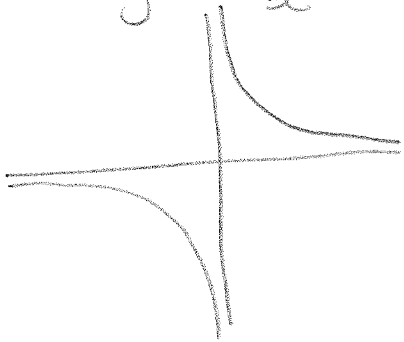
$\longrightarrow$ $\longleftarrow$



Sketch graph of $y = \frac{1}{x} + 2$, showing the points of intersection with the coordinate axis and stating the equations of any asymptotes.

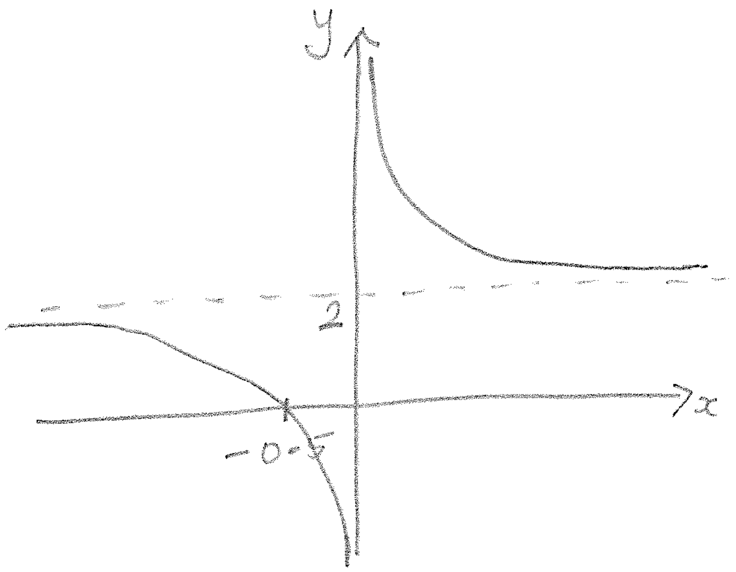
$y = \frac{1}{x} + 2$

Think $y = \frac{1}{x}$



$y = \frac{1}{x} + 2$

moves $\frac{1}{x}$ up 2 units



Try when $x = 0.01$

$\frac{1}{0.01} + 2$

$= 102$

Try when $x = -0.01$

$\frac{1}{-0.01} + 2$

-98

where does $\frac{1}{x} + 2$ cuts y axis

$\frac{1}{0} + 2$ not possible

where does it cut the x axis

$0 = \frac{1}{x} + 2$

$0 = \frac{1}{-0.5} + 2$

$$f(x) = (x+3)(x-1)^2$$

(a) Sketch the curve $y = f(x)$, showing the points of intersection with the coordinate axis. **(3)**

(b) Find the equation of $y = f(x+2)$ in the form $y = (x+a)(x+b)^2$ **(2)**

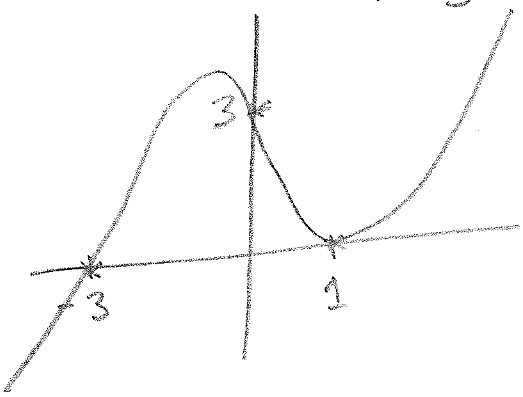
$f(x) = (x+3)(x-1)^2$

cuts x axis when $x = -3$ and $x = 1$ repeated

cuts y axis when $x = 0$

$f(0) = (3)(-1)^2$

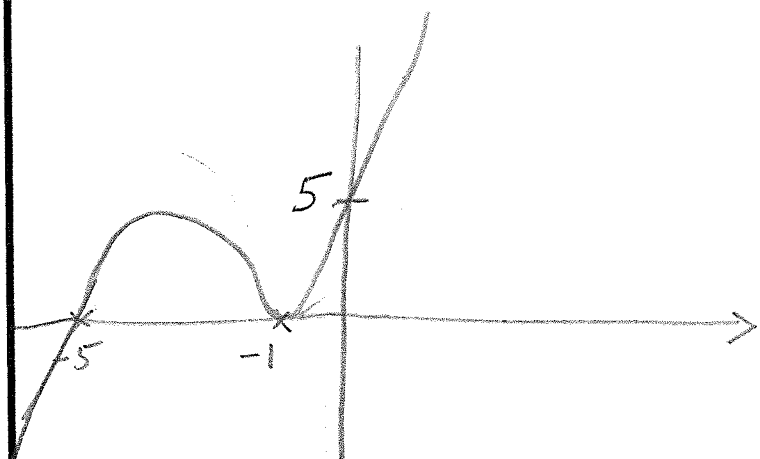
$= 3$



x^3 shape

$f(x+2) = (x+5)(x+1)^2$

$f(x+2)$ is Beyoncé Transformation



"to the left"

Given that the constants p and q are such that $p > q > 0$. sketch each of the following graphs showing the coordinates of any points of intersection with the coordinate axes.

a $y = (x - p)(x - q)^2$

cuts x axis

$x = p$ $\quad$ $x = q$

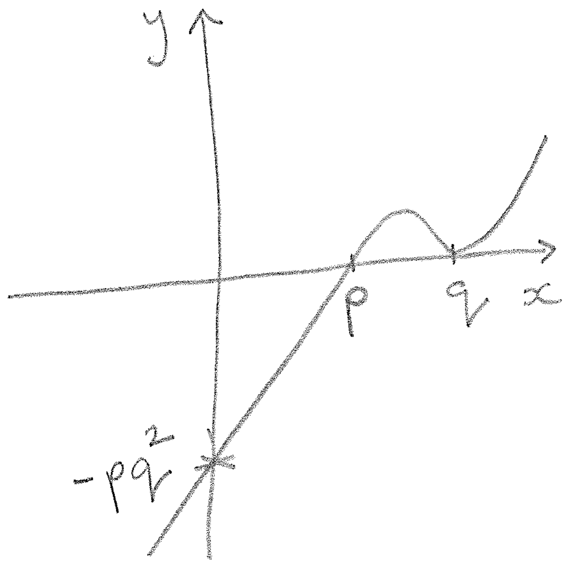
$(p, 0)$ and repeated at $(q, 0)$

cuts y axis when $x = 0$

$y = (-p)(-q)^2$

$= -pq^2$

$(0, -pq^2)$



b $y = (x - p)(x^2 - q^2)$

$y = (x-p)(x+q)(x-q)$

Diff of 2 squares

cuts x axis at

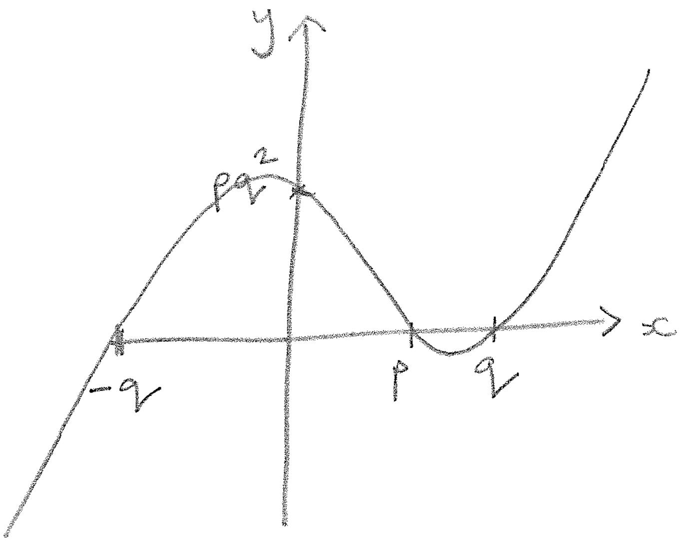
$(p, 0)$ $(q, 0)$ $(-q, 0)$

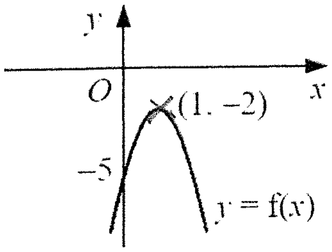
When $x = 0$

cuts y axis

$y = (-p)(-q^2)$

$y = pq^2$ $(0, pq^2)$





The diagram shows the curve with equation $y = f(x)$ which has a turning point at $(1, -2)$ and crosses the y-axis at the point $(0, -5)$.

Given that $f(x)$ is a quadratic function, find an expression for $f(x)$.

$f(x) = ax^2 + bx + c$

$f(1) = \boxed{a + b + c = -2}$ $\quad a+b = 3$

$f(0) = \boxed{c = -5}$

$f(x) = ax^2 + bx - 5$

$f'(x) = 2ax + b$

But $f'(1) = 0$ $\quad \boxed{2a + b = 0}$

$2a + b = 0$
$a + b = 3$
$a = -3$ $\quad a = -3$

$a + b = 3$ $\quad b = 6$

$-3 + b = 3$ $\quad c = -5$

$-3x^2 + 6x - 5 = f(x)$

check

$-x^2$ ∩ shape ✓

when $f(0) = -5$ ✓

$f(1) = -2$ ✓

$f'(x) = -6x + 6$

$f'(x) = 0$

when $x = 1$ ✓

a Find the coordinates of the points where the straight line $y = x + 6$ meets the curve $y = x^3 - 4x^2 + x + 6$.

b Given that

$$x^3 - 4x^2 + x + 6 \equiv (x + 1)(x - 2)(x - 3),$$

sketch the straight line $y = x + 6$ and the curve $y = x^3 - 4x^2 + x + 6$ on the same diagram, showing the coordinates of the points where the curve crosses the coordinate axes.

$$\left.\begin{aligned} y &= x^3 - 4x^2 + x + 6 \\ y &= x + 6 \end{aligned}\right] \text{ solve simultaneously}$$

$$x + 6 = x^3 - 4x^2 + x + 6$$

$$0 = x^3 - 4x^2$$

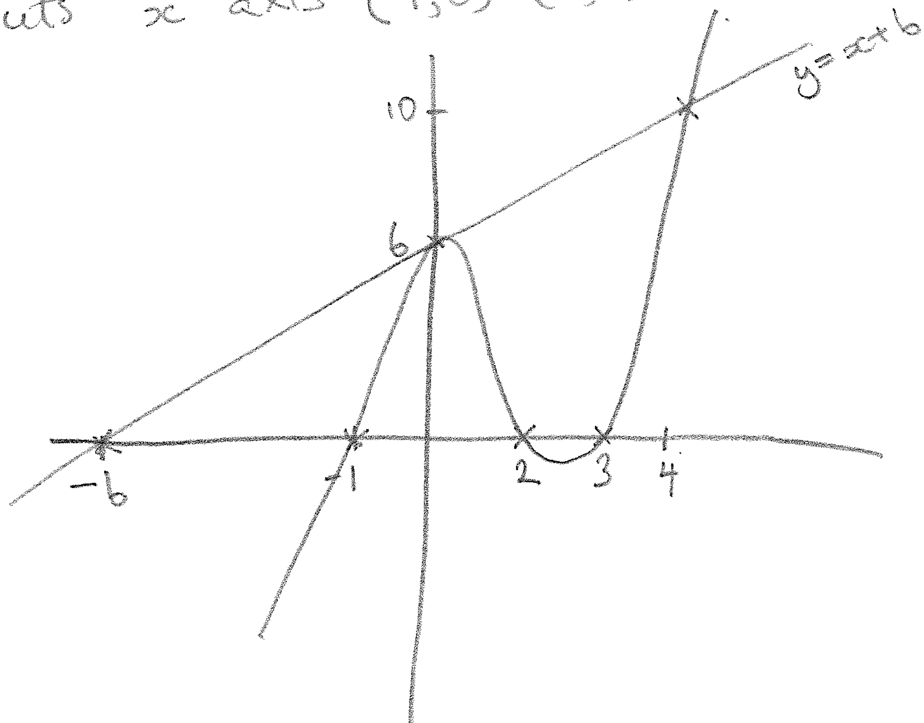
$$0 = x^2(x - 4)$$

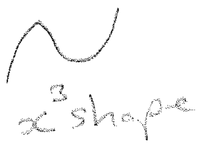
$x = 0$ repeated, $y = 6$, $(0,6)$

$x = 4$, $y = 10$, $(4,10)$

$$x^3 - 4x^2 + x + 6 \equiv (x+1)(x-2)(x-3)$$

cuts x axis $(-1,0)$ $(2,0)$ $(3,0)$





Find the set of values of the constant a for which the line $y = 2 - 5x$ intersects the curve $y = x^2 + ax + 18$ at two points.

$y = 2 - 5x$ $\quad$ $y = x^2 + ax + 18$

solve simultaneously

$2 - 5x = x^2 + ax + 18$

$0 = x^2 + (a+5)x + 16$

Intersects when " $b^2 - 4ac > 0$ "

discriminent > 0

$(a+5)^2 - 4(1)(16) > 0$

$a^2 + 10a + 25 - 64 > 0$

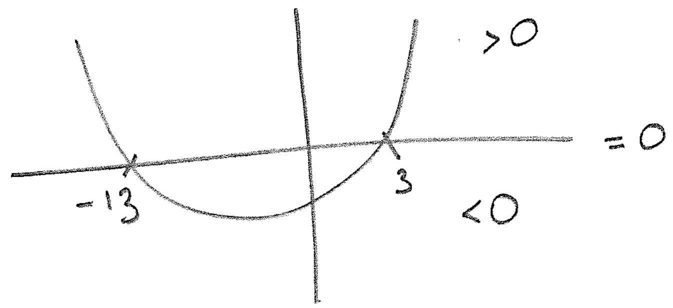
$a^2 + 10a - 39 > 0$

$(a + 13)(a - 3) > 0$

Look at the graph $\quad$ a^2 U shape

Happy shape

$a = -13 \quad a = 3$



Discriminent > 0 when $a < -13$ & $a > 3$

Answer $a < -13$ & $a > 3$

(a) Sketch on the same diagram the curves $y = x^2 + 5x$ and $y = -\frac{1}{x}$ **(4)**

(b) State, giving a reason, the number of real solutions to the equation $x^2 + 5x + \frac{1}{x} = 0$ **(2)**

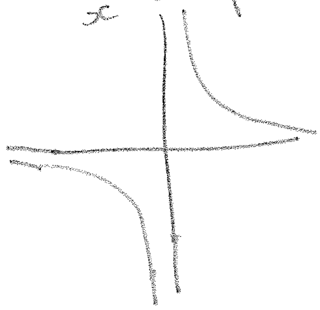
$y = x^2 + 5x$ $\qquad$ $y = -\frac{1}{x}$

$0 = y = x(x+5)$

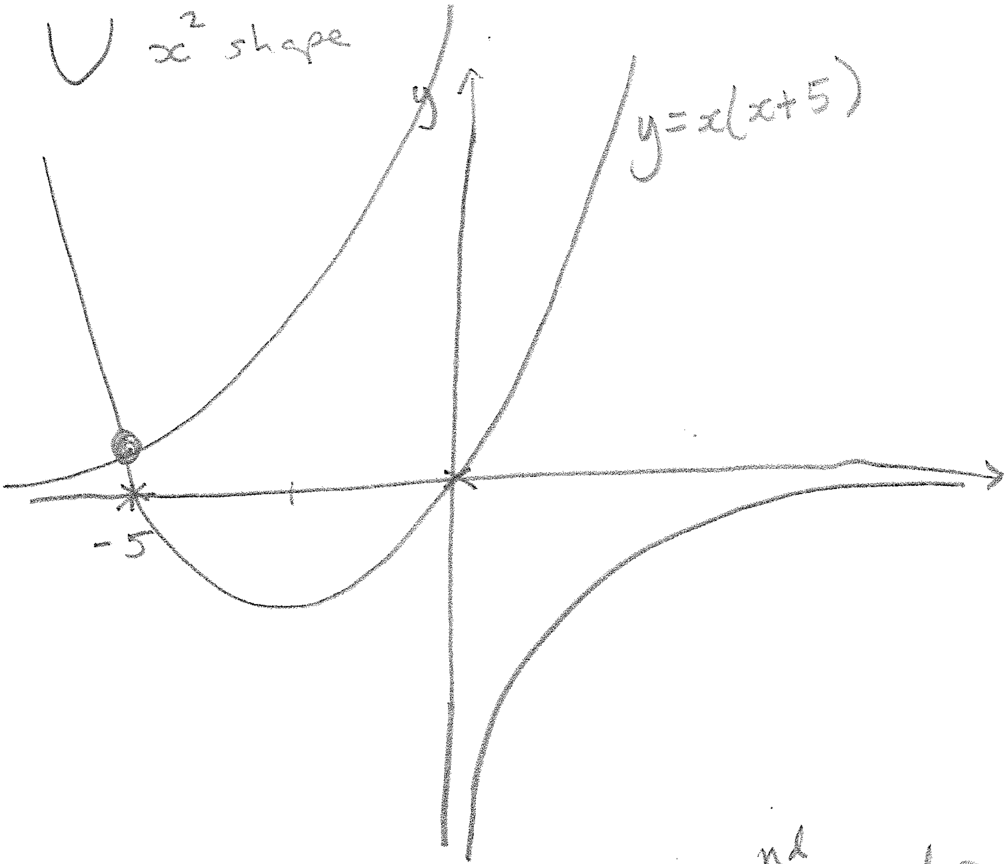
$x = 0$ $\qquad$ $x = -5$

$(0,0)$ $\qquad$ $(-5,0)$

$\frac{1}{x}$ shape



x^2 shape



only one solution in 2^{nd} quadrant

$$x^2 + 5x + \frac{1}{x} = 0$$

$$x^2 + 5x = -\frac{1}{x}$$

$$x^3 + 5x^2 + 1 = 0$$

cubic

a Find the coordinates of the turning point of the curve $y = x^2 + 2x - 3$.

b By sketching two suitable graphs on the same set of axes, show that the equation

$$x^2 + 2x - 3 - \frac{1}{x} = 0$$

has one positive and two negative real roots.

a) could use $\frac{dy}{dx}$ then $\frac{dy}{dx} = 0$

but could use "completing the square"

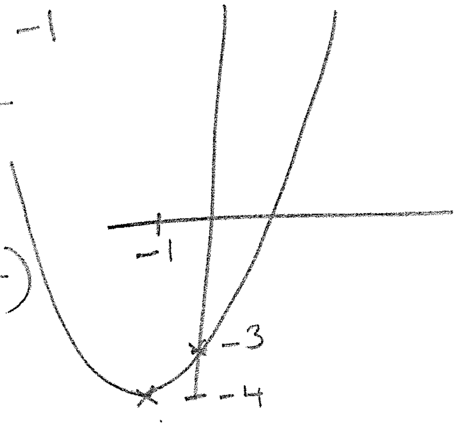
$y = x^2 + 2x - 3$

$y = (x+1)^2 - 3 - 1$

$y = (x+1)^2 - 4$

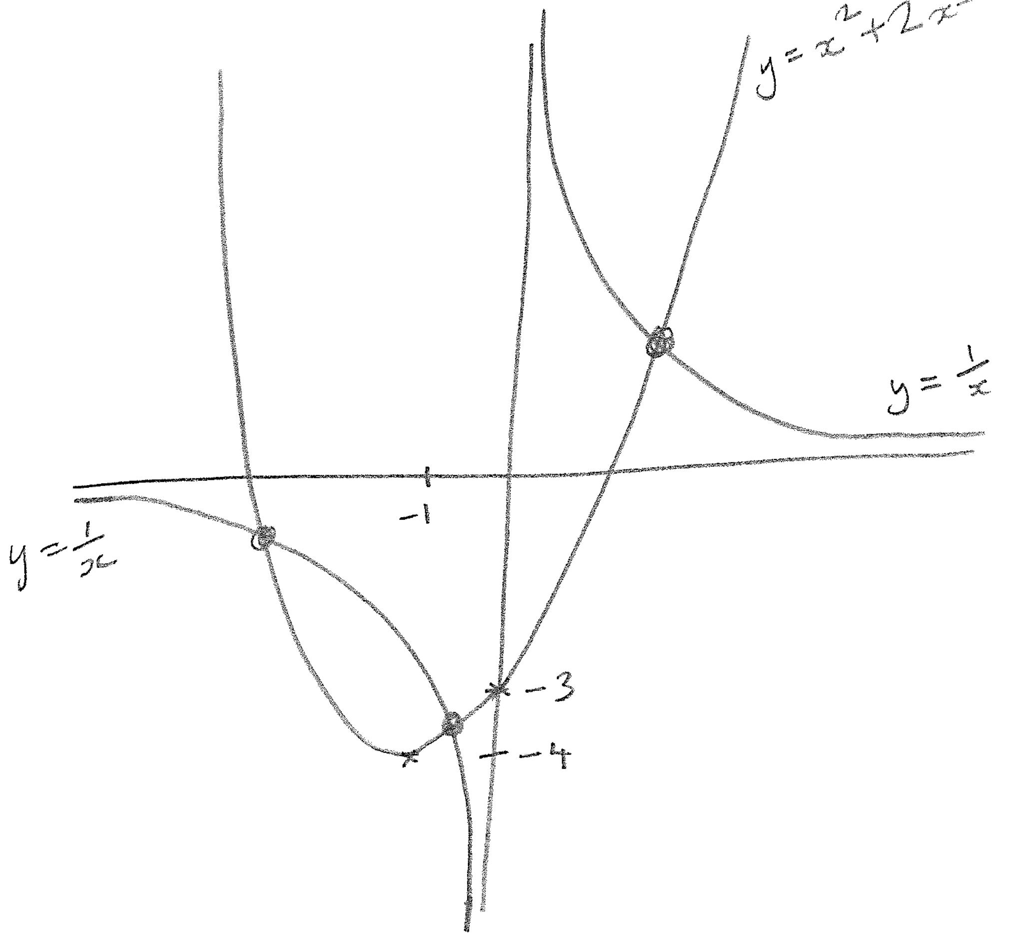
x^2 happy graph

Turning point at $(-1, -4)$



$x^2 + 2x - 3 - \frac{1}{x} = 0$

$x^2 + 2x - 3 = \frac{1}{x}$



a Find the coordinates of the points where the straight line $y = x + 6$ meets the curve $y = x^3 - 4x^2 + x + 6$.

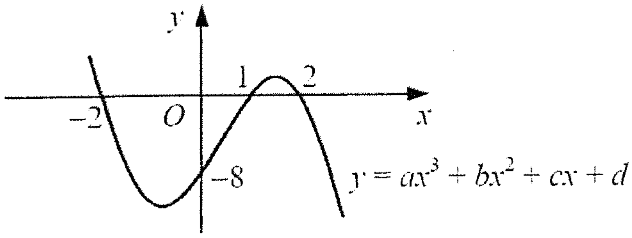
b Given that

$$x^3 - 4x^2 + x + 6 \equiv (x + 1)(x - 2)(x - 3),$$

sketch the straight line $y = x + 6$ and the curve $y = x^3 - 4x^2 + x + 6$ on the same diagram, showing the coordinates of the points where the curve crosses the coordinate axes.

Sorry!

repeated!!



The diagram shows the curve with equation $y = ax^3 + bx^2 + cx + d$.

Given that the curve crosses the y-axis at the point $(0, -8)$ and crosses the x-axis at the points $(-2, 0)$, $(1, 0)$ and $(2, 0)$, find the values of the constants a, b, c and d.

cuts at $(-2,0)$ $(1,0)$ and $(2,0)$

must be $(x+2)(x-1)(x-2)$

$f(x) = ?(x+2)(x-1)(x-2)$

$f(0) = ?\,(2)(-1)(-2)$

$f(0) = 4 \times ?$

so $? = -2$ because $f(0) = -8$

so

$f(x) = -2(x+2)(x-1)(x-2)$

$= -2(x+2)(x^2-3x+2)$

$= -2(x^3-3x^2+2x+2x^2-6x+4)$

$= -2(x^3-x^2-4x+4)$

$f(x) = -2x^3+2x^2+8x-8$

Remember to check the roots on your fancy white calculator