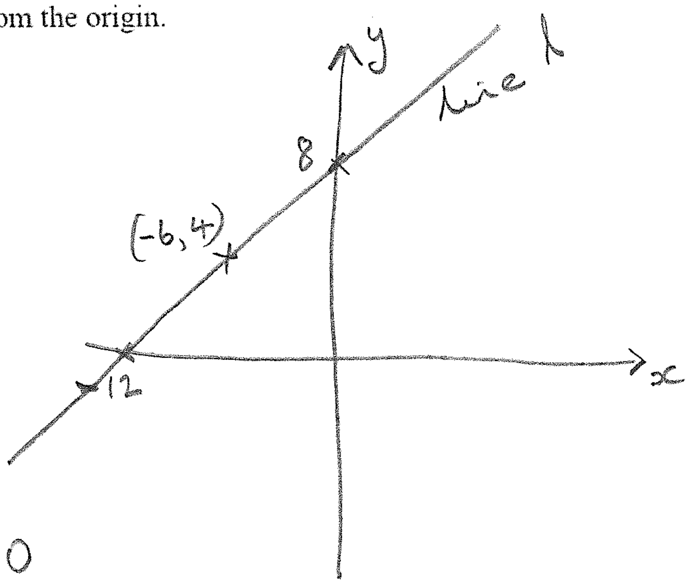
The straight line l has equation $2x - 3y + 24 = 0$ and meets the coordinate axis at the points A and B.

Find the distance of the midpoint of AB from the origin.

Give your answer in the form $k\sqrt{13}$



$2x - 3y + 24 = 0$

cuts y axis when $x = 0$

$0 - 3y + 24 = 0$

$3y = 24$

$y = 8$

cuts x axis when $y = 0$

$2x - 0 + 24 = 0$

$2x = -24$

$x = -12$

Mid point of $(0,8)$ & $(-12,0)$ is $(-6,4)$

Find distance from $(0,0)$ to $(-6,4)$

$\sqrt{6^2 + 4^2}$

$= \sqrt{36 + 16}$

$= \sqrt{52}$

$= \sqrt{4} \times \sqrt{13}$

$= 2\sqrt{13}$

The points A and B have coordinates $(-1, k+2)$ and $(2k-3, 8)$ where k is a constant.

Given the gradient of AB is $\frac{1}{3}$

(a) Show that $k = 4$ **(2)**

(b) Find the equation of the line the passes through A and B. **(3)**

(c) Find the equation of the perpendicular bisector of A and B.
Give your answer in the form $ax + by + c = 0$ **(4)**

$A(-1, k+2) \quad B(2k-3, 8)$

Gradient of $AB = \dfrac{\text{Change in } y}{\text{Change in } x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{8-(k+2)}{2k-3+1}$

$\dfrac{1}{3} = \dfrac{6-k}{2k-2}$

$(2k-2) = 3(6-k)$

$2k - 2 = 18 - 3k$

$5k = 20$ ✓

$k = 4$

$A = (-1, 6) \quad B = (5, 8)$

$y = \frac{1}{3}x + c$

$6 = \frac{1}{3}(-1) + c$

$6\frac{1}{3} = c$

$y = \frac{1}{3}x + \frac{19}{3}$

perp. bisector gradient -3

What is mid-point $(2, 7)$

$y = -3x + c$

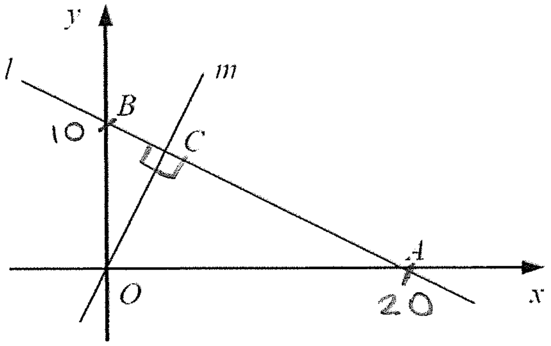
$7 = -3(2) + c$

$13 = c$

$y = -3x + 13$

$3x + y - 13 = 0$

Ans $3x + y - 13 = 0$



The diagram shows the straight line l with equation $x + 2y - 20 = 0$ and the straight line m which is perpendicular to l and passes through the origin O.

a Find the coordinates of the points A and B where l meets the x-axis and y-axis respectively. **(2)**

Given that l and m intersect at the point C,

b find the ratio of the area of triangle OAC to the area of triangle OBC. **(5)**

Find point A when $y=0$

$x + 0 - 20 = 0$

$x = 20$

Ans $A = (20, 0)$

Find point B when $x=0$

$0 + 2y - 20 = 0$

$y = 10$

$B = (0, 10)$

Gradient of AB $= -1/2$

Gradient of line m $= +2$

Equation of m $y = 2x + 0$ passes through the origin

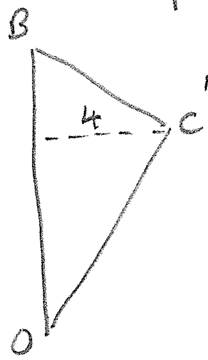
where do $x + 2y - 20 = 0$ cut the line $y = 2x$

$x + 2(2x) - 20 = 0$

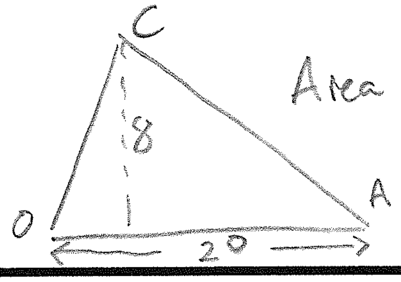
$5x - 20 = 0$

$x = 4$

C is point $(4, 8)$



Area BCO = 20



Area OAC = 80

Ratio

area OAC : area OBC

80 : 20

4 : 1

The straight line l passes through the points A $(1,\ 2\sqrt{3})$ and B $(\sqrt{3}, 6)$.

a Find the gradient of l in its simplest form.

b Show that l also passes through the origin.

c Show that the straight line which passes through A and is perpendicular to l has equation

$$x + 2\sqrt{3}\,y - 13 = 0.$$

$A = (1, 2\sqrt{3})$ $\quad B = (\sqrt{3}, 6)$

$$\text{Gradient} = \frac{6 - 2\sqrt{3}}{\sqrt{3} - 1} = \frac{(6 - 2\sqrt{3})}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)} = \frac{6\sqrt{3} + 6 - 6 - 2\sqrt{3}}{2}$$

$$= \frac{4\sqrt{3}}{2} = 2\sqrt{3}$$

Ans Gradient $= 2\sqrt{3}$

$y = mx + c$

Find c. Substitute in point $(1, 2\sqrt{3})$

$2\sqrt{3} = 2\sqrt{3}(1) + c$

$2\sqrt{3} = 2\sqrt{3} + c$

$0 = c$

$y = 2\sqrt{3}x + 0$ line l

perpendicular to line l means gradient $\frac{-1}{2\sqrt{3}}$

$y = \frac{-1}{2\sqrt{3}}x + c$

passes through $(1, 2\sqrt{3})$

$2\sqrt{3} = \frac{-1}{2\sqrt{3}}(1) + c$

$2\sqrt{3} + \frac{1}{2\sqrt{3}} = c$

$\frac{12 + 1}{2\sqrt{3}} = c$

$y = \frac{-1}{2\sqrt{3}}x + \frac{13}{2\sqrt{3}}$

$2\sqrt{3}y = -x + 13$

$x + 2\sqrt{3}y - 13 = 0$

The vertices of a triangle are the points P (3, c), Q (9, 2) and R (3c, 11) where c is a constant.

Given that $\angle PQR = 90°$,

a find the value of c, **(5)**

b show that the length of PQ is $k\sqrt{10}$, where k is an integer to be found, **(3)**

c find the area of triangle PQR. **(4)**

$P\hat{Q}R = 90°$

Gradient of PQ × Gradient of QR = −1

$$\frac{2-c}{6} \times \frac{9}{3c-9} = -1$$

$$\frac{9(2-c)}{6(3c-9)} = -1$$

$$18 - 9c = 54 - 18c$$

$$9c = 36$$

$$c = 4$$

Ans $c = 4$

$P = (3, 4)$ $Q = (9, 2)$ $R = (12, 11)$

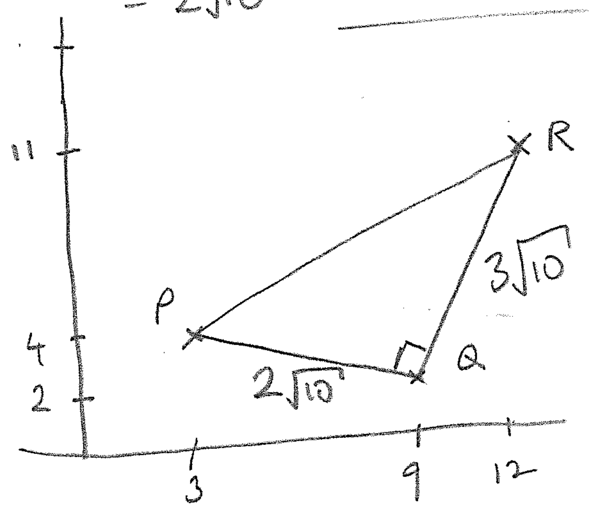
$$PQ = \sqrt{6^2 + 2^2}$$

$$= \sqrt{36 + 4}$$

$$= \sqrt{40}$$

$$= \sqrt{4}\sqrt{10}$$

$$= 2\sqrt{10}$$



$$QR = \sqrt{3^2 + 9^2}$$

$$QR = \sqrt{90}$$

$$QR = \sqrt{9}\sqrt{10}$$

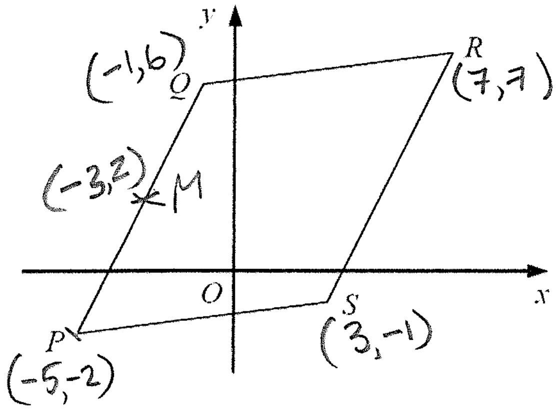
$$QR = 3\sqrt{10}$$

$$\text{Area} = \frac{\text{base} \times \text{height}}{2}$$

because it is right-angled

$$\text{Area} = \frac{2\sqrt{10} \times 3\sqrt{10}}{2}$$

$$= 30$$



The points $P(-5, -2)$, $Q(-1, 6)$, $R(7, 7)$ and $S(3, -1)$ are the vertices of a parallelogram as shown in the diagram above.

a Find the length of PQ in the form $k\sqrt{5}$, where k is an integer to be found. **(3)**

b Find the coordinates of the point M, the mid-point of PQ. **(2)**

c Show that MS is perpendicular to PQ. **(4)**

d Find the area of parallelogram $PQRS$. **(4)**

$PQ = \sqrt{4^2 + 8^2}$

$PQ = \sqrt{16 + 64}$

$PQ = \sqrt{80}$

$PQ = \sqrt{16} \times \sqrt{5}$

$PQ = 4\sqrt{5}$

Ans $k = 4$

mid-point of PQ $(-3,2)$

Show MS is perp to PQ

Gradient of MS $= -\frac{3}{6} = \frac{-1}{2}$ Gradient of PQ $= \frac{8}{4} = +2$

$(-\frac{1}{2}) \times (+2) = -1$ so yes perpendicular

Many ways to do this but easiest is

Area ▱ $=$ Base PQ $\times$ Perp height MS

$= 4\sqrt{5} \times (\sqrt{6^2 + 3^2})$

$= 4\sqrt{5} \times \sqrt{45}$

$= 4\sqrt{5} \times \sqrt{9} \times \sqrt{5}$

$= 4 \times 3 \times 5$

$= 60$

The point A has coordinates $(-2, 7)$ and the point B has coordinates $(4, p)$.

The point M is the mid-point of AB and has coordinates $(q, \frac{9}{2})$.

a Find the values of the constants p and q. **(3)**

b Find the equation of the straight line perpendicular to AB which passes through the point A. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(5)**

$A = (-2, 7) \quad B = (4, p)$

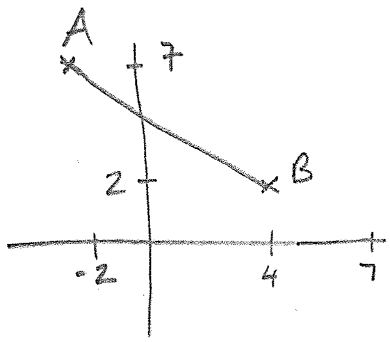
mid-pt $(q, \frac{9}{2})$

$\frac{4 + (-2)}{2} = q \qquad \frac{7 + p}{2} = \frac{9}{2}$

$\frac{2}{2} = q \qquad 7 + p = 9$

$1 = q \qquad p = 2$

$A = (-2, 7) \quad B = (4, 2)$



Grad of AB $= \frac{-5}{6}$

Perp gradient $= \frac{6}{5}$

$y = mx + c$

$y = \frac{6}{5}x + c$

Substitute in $(-2, 7)$

$7 = \frac{6}{5}(-2) + c$

$7 = \frac{-12}{5} + c$

$7 + 2\frac{2}{5} = c$

$9\frac{2}{5} = c$

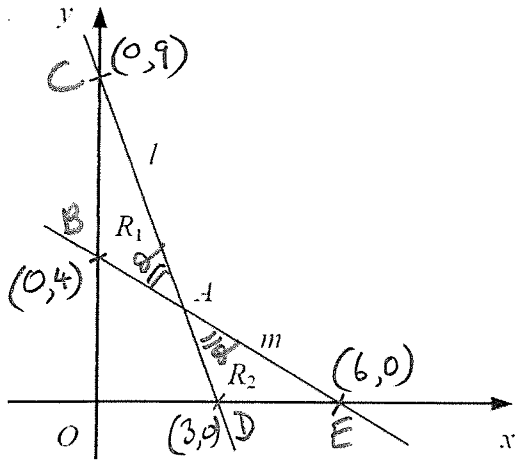
$y = \frac{6}{5}x + 9\frac{2}{5}$

$y = \frac{6}{5}x + \frac{47}{5}$

$5y = 6x + 47$

Ans

$6x - 5y + 47 = 0$



$2x + 3y - 12 = 0$

cuts 6,0)

Find Areas by using "$A = \frac{1}{2}ab\sin C$"

They share the same angle indicated

The line l with equation $3x + y - 9 = 0$ intersects the line m with equation $2x + 3y - 12 = 0$ at the point A as shown in the diagram above.

a Find, as exact fractions, the coordinates of the point A.

The region R_1 is bounded by l, m and the y-axis.

The region R_2 is bounded by l, m and the x-axis.

b Show that the ratio of the area of R_1 to the area of R_2 is $25 : 18$

solve simultaneously

$3x + y - 9 = 0$

$2x + 3y - 12 = 0$

$2x + 3(9 - 3x) - 12 = 0$

$2x + 27 - 9x - 12 = 0$

$15 - 7x = 0$

$15 = 7x$

$2\frac{1}{7} = \frac{15}{7} = x$

$y = 9 - 3x$

$y = 9 - 3\left(\frac{15}{7}\right)$

$y = 9 - \frac{45}{7}$

$y = \frac{18}{7} = 2\frac{4}{7}$

$\left(\frac{15}{7}, \frac{18}{7}\right)$

You must find exact length of AB and AC

$AB = \sqrt{\left(\frac{15}{7}\right)^2 + \left(\frac{10}{7}\right)^2}$

$= \sqrt{\frac{225}{49} + \frac{100}{49}}$

$= \sqrt{\frac{325}{49}}$

$AC = \sqrt{\left(\frac{15}{7}\right)^2 + \left(\frac{45}{7}\right)^2}$

$= \sqrt{\frac{225}{49} + \frac{2025}{49}}$

$= \sqrt{\frac{2250}{49}}$

$AD = \sqrt{\left(\frac{6}{7}\right)^2 + \left(\frac{18}{7}\right)^2}$

$AD = \sqrt{\frac{360}{49}}$

$AE = \sqrt{\left(\frac{27}{7}\right)^2 + \left(\frac{18}{7}\right)^2}$

$= \sqrt{\frac{1053}{49}}$

Then Area R_1 : Area R_2

$\frac{1}{2}\left(\sqrt{\frac{325}{49}}\right)\left(\sqrt{\frac{2250}{49}}\right)\sin\theta : \frac{1}{2}\left(\sqrt{\frac{360}{49}}\right)\left(\sqrt{\frac{1053}{49}}\right)\sin\theta$

$75\sqrt{130} : 54\sqrt{130}$

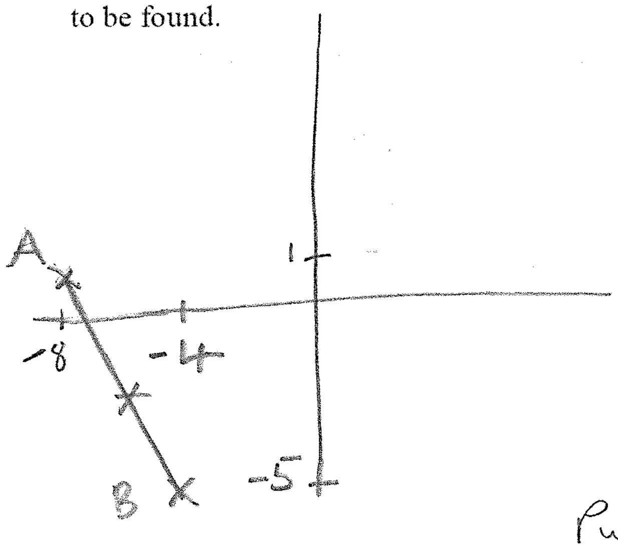
$75 : 54$

$25 : 18$

The point A has coordinates $(-8, 1)$ and the point B has coordinates $(-4, -5)$.

a Find the equation of the straight line passing through A and B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

b Show that the distance of the mid-point of AB from the origin is $k\sqrt{10}$ where k is an integer to be found.



Gradient of AB $= \frac{-6}{4}$

$= -\frac{3}{2}$

$y = mx + c$

$y = -\frac{3}{2}x + c$

Put in point $A(-8,1)$ to find C

$1 = -\frac{3}{2}(-8) + c$

$1 = +12 + c$

$-11 = c$

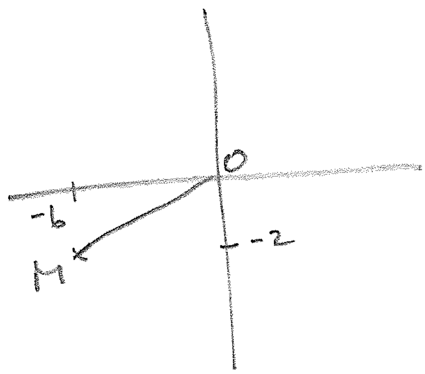
Ans $y = -\frac{3}{2}x - 11$ ✓

Mid-point of AB

$M = \left(\frac{(-8)+(-4)}{2}, \frac{1+(-5)}{2}\right)$

$M = (-6, -2)$

Distance from $(-6, -2)$ to origin



Pythagoras'

$\sqrt{6^2 + 2^2}$

$= \sqrt{36 + 4}$

$= \sqrt{40}$

$= \sqrt{4}\sqrt{10}$

$= 2\sqrt{10}$

Answer $k = 2$

The straight line l_1 has gradient $\frac{1}{3}$ and passes through the point with coordinates $(-3, 4)$.

a Find the equation of l_1 in the form $ax + by + c = 0$, where a, b and c are integers.

The straight line l_2 has the equation $5x + py - 2 = 0$ and intersects l_1 at the point with coordinates $(q, 7)$.

b Find the values of the constants p and q.

$y = \frac{1}{3}x + c$

Put in point $(-3, 4)$

$4 = \frac{1}{3}(-3) + c$

$4 = -1 + c$

$5 = c$

$y = \frac{1}{3}x + 5$ get rid of fraction

$3y = x + 15$

$0 = x - 3y + 15$

Answer $x - 3y + 15 = 0$

2 lines $x - 3y + 15 = 0$ and $5x + py - 2 = 0$

intersect at point $(q, 7)$

Solve simultaneously

$x - 3y + 15 = 0$

$x = 3y - 15$

$5x + py - 2 = 0$

$5(3y - 15) + py - 2 = 0$

$15y - 75 + py - 2 = 0$

But $y = 7$

$x = 3y - 15$

$x = 3 \times 7 - 15$

$x = 6$

$q = 6$ $(6, 7)$

Put in $x = 6$, $y = 7$ into

$5x + py - 2 = 0$

$5(6) + 7p - 2 = 0$

$30 + 7p - 2 = 0$

$7p = -28$

$p = -4$

Ans $p = -4$ & $q = 6$

The straight line l passes through the point A (1, −3) and the point B (7, 5).

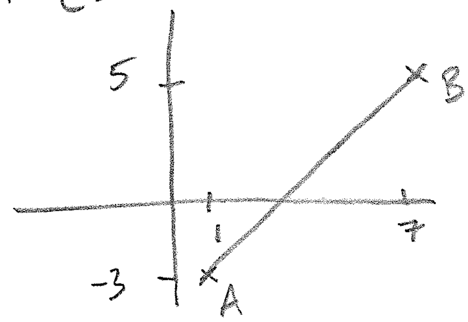
a Find an equation of line l. **(3)**

The line m has the equation $4x + y - 17 = 0$ and intersects l at the point C.

b Show that C is the mid-point of AB. **(4)**

c Show that the straight line perpendicular to m which passes through the point C also passes through the origin. **(4)**

$A = (1,-3) \quad B = (7,5)$



Gradient $= \frac{8}{6} = \frac{4}{3}$

$y = \frac{4}{3}x + c$

Put in point $(7,5)$

$5 = \frac{4}{3}(7) + c$

$5 = \frac{28}{3} + c$

$-\frac{13}{3} = c$ Ans $y = \frac{4}{3}x - \frac{13}{3}$

$3y = 4x - 13$

Intersects – Solve simultaneously

$3y = 4x - 13$ and $4x + y - 17 = 0$

$y = 17 - 4x$

$3(17 - 4x) = 4x - 13$

$51 - 12x = 4x - 13$

$64 = 16x$

$4 = x$

$y = 17 - 4(4)$

$y = 1$

Ans $C = (4,1)$

Mid-point of $AB = \left(\frac{1+7}{2}, \frac{-3+5}{2}\right) = (4,1)$ Proved

$4x + y - 17 = 0$

$y = -4x + 17$

Gradient of m is -4

Gradient of perp. is $\frac{1}{4}$

$y = \frac{1}{4}x + c$

Through point $(4,1)$

$1 = \frac{1}{4}(4) + c$

$1 = 1 + c$

$0 = c$

so line is $y = \frac{1}{4}x + 0$

so intercepts the axes at origin

The straight line l has gradient $\frac{1}{2}$ and passes through the point with coordinates (2, 4).

a Find the equation of l in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

The straight line m has the equation $y = 2x - 6$.

b Find the coordinates of the point where line m intersects line l. **(3)**

c Show that the quadrilateral enclosed by line l, line m and the positive coordinate axes is a kite. **(4)**

$y = \frac{1}{2}x + c$

Through point (2, 4)

$4 = \frac{1}{2}(2) + c$

$4 = 1 + c$

$3 = c$

$y = \frac{1}{2}x + 3$

Get rid of fractions

$2y = x + 6$

Ans $0 = \underline{x - 2y} + 6$

Another line $y = 2x - 6$ intersects $0 = x - 2y + 6$

so solve simultaneously by putting $2x - 6$ into here

$0 = x - 2(2x - 6) + 6$

$0 = x - 4x + 12 + 6$

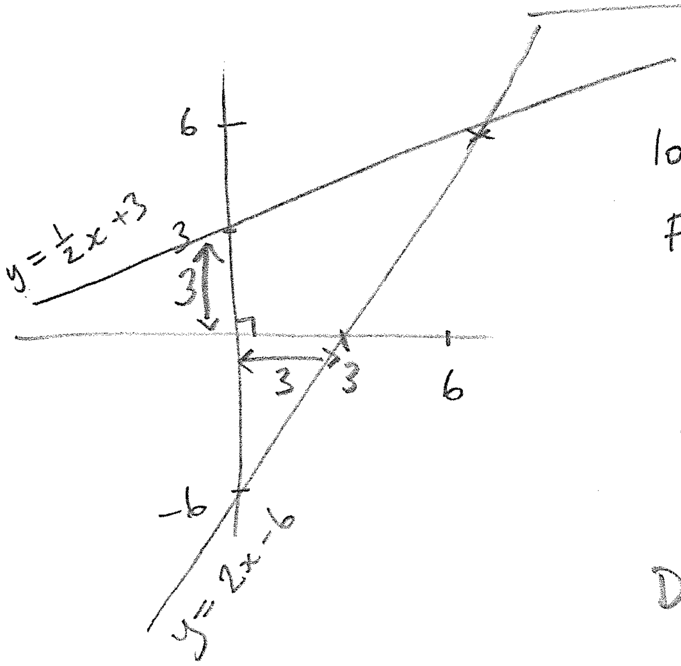
$3x = 18$

$x = 6$

Then $y = 2(6) - 6$

$y = 12 - 6$

$y = 6$ Ans (6, 6)



Prove a kite by looking at lengths of 4 sides

Find distance from (0, 3) to (6, 6)

$\sqrt{6^2 + 3^2}$

$= \sqrt{36 + 9}$

$= \sqrt{45}$

Distance from (3, 0) to (6, 6)

$\sqrt{3^2 + 6^2}$

$= \sqrt{9 + 36}$

$= \sqrt{45}$