The points A, B and C lie on a plane so that

$$\overrightarrow{AB} = 2\mathbf{i} + 7\mathbf{j} \quad \text{and} \quad \overrightarrow{AC} = 4\mathbf{i} - 5\mathbf{j},$$

where $\mathbf{i}$ and $\mathbf{j}$ are mutually perpendicular unit vectors lying on the same plane.

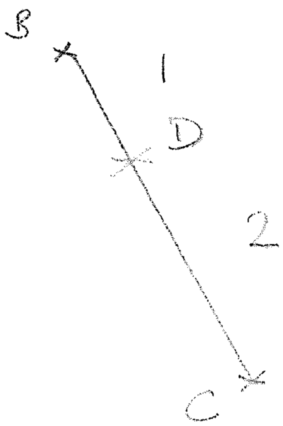
The point D lies on the straight line segment BC, so that $|BD|:|DC| = 1:2$.

a) Determine a simplified expression, in terms of $\mathbf{i}$ and $\mathbf{j}$, for $\overrightarrow{BD}$.

b) Show that the $|\overrightarrow{AD}|$ is approximately 4 units.

$\overrightarrow{AB} = 2i + 7j \qquad \overrightarrow{AC} = 4i - 5j$

$\overrightarrow{BC} = 2i - 12j \qquad \overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$



$\overrightarrow{BC} = 2i - 12j$

BD : DC is 1:2

$\overrightarrow{BD} = \frac{1}{3}\overrightarrow{BC} = \frac{1}{3}(2i - 12j)$

$= \frac{2}{3}i - 4j$

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD} = 2i + 7j + \frac{2}{3}i - 4j = 2\frac{2}{3}i + 3j$

$|\overrightarrow{AD}| = \sqrt{(2\frac{2}{3})^2 + (3)^2} = \sqrt{(\frac{8}{3})^2 + 9} = \sqrt{\frac{64}{9} + 9}$

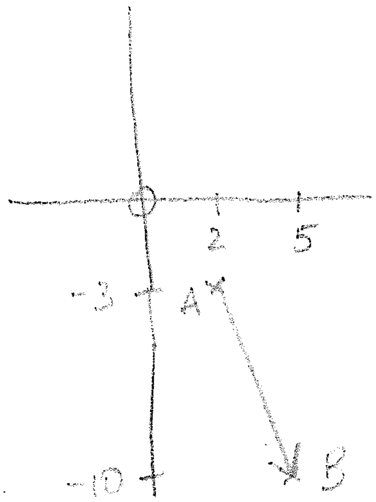
$\overrightarrow{AD} = \sqrt{16\frac{1}{9}}$

$= 4.01$

Relative to a fixed origin O, the point A has coordinates $(2,-3)$.

The point B is such so that $\overrightarrow{AB} = 3\mathbf{i} - 7\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are mutually perpendicular unit vectors lying on the same plane.

Determine the distance of B from O.



$A = (2,-3)$

$\overrightarrow{AB} = 3i - 7j$

$B = (5,-10)$

$\text{Distance } OB = \sqrt{5^2 + (-10)^2} = \sqrt{25 + 100}$

$\text{Distance } OB = \sqrt{125}$

The following information is given for four points which lie on the same plane.

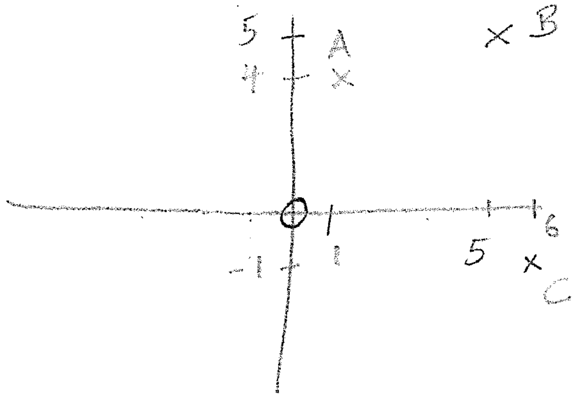
$$\overrightarrow{OA} = \mathbf{i} + 4\mathbf{j}, \quad \overrightarrow{OB} = 5\mathbf{i} + 5\mathbf{j} \quad \text{and} \quad \overrightarrow{CB} = -\mathbf{i} + 6\mathbf{j},$$

$\overrightarrow{BC} = i - 6j$

a) Find the vector $\overrightarrow{AB}$ and hence state its length

b) Determine the length of $\overrightarrow{AC}$.

c) Calculate the size of the angle ABC.



$\overrightarrow{AB} = 4i + j$ ✓

$|\overrightarrow{AB}| = \sqrt{4^2 + 1^2}$
$= \sqrt{17}$

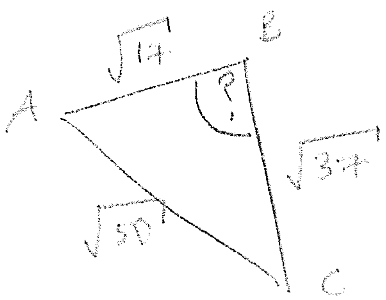
Point C is $(6,-1)$

$\overrightarrow{AC} = 5i - 5j$

$|\overrightarrow{AC}| = \sqrt{50}$

$\overrightarrow{BC} = i - 6j$

$\overrightarrow{BC} = \sqrt{37}$



use Cosine Rule

$b^2 = a^2 + c^2 - 2ac \cos B$

$50 = 37 + 17 - 2\sqrt{17}\sqrt{37} \cos \hat{B}$

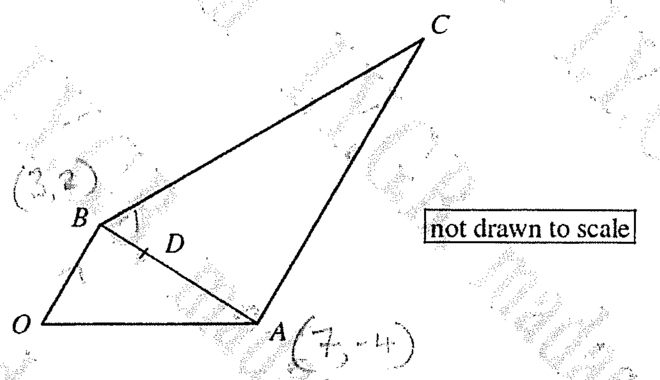
$2\sqrt{17}\sqrt{37} \cos \hat{B} = 37 + 17 - 50$

$\cos B = \dfrac{4}{2\sqrt{17}\sqrt{37}} \Rightarrow B = 85.4^\circ$

$\vec{OB} = \sqrt{13}$

$\vec{BA} = \sqrt{52}$

$\vec{OA} = \sqrt{65}$



The figure above shows a trapezium $OBCA$ where OB is parallel to AC.

The point D lies on BA so that $BD : DA = 1:2$.

It is further given that $\overline{OA} = 7\mathbf{i} - 4\mathbf{j}$, $\overline{OB} = 3\mathbf{i} + 2\mathbf{j}$ and $\overline{AC} = 2\,\overline{OB}$, where $\mathbf{i}$ and $\mathbf{j}$ are mutually perpendicular unit vectors lying on the same plane.

a) Determine simplified expressions, in terms of $\mathbf{i}$ and $\mathbf{j}$, for each of the vectors $\overline{OC}$, $\overline{AB}$, $\overline{AD}$ and $\overline{OD}$.

b) Deduce, showing your reasoning, that O, D and C are collinear and state the ratio of $OC : OD$.

use Pythagoras

c) Show that $\measuredangle OBA = 90°$ and hence find the area of the trapezium $OBCA$.

d) State the size of the angle $\measuredangle ABC$. use Cosine Rule

$\vec{OB} = 3i + 2j$ $\qquad$ $\vec{OC} = \vec{OA} + \vec{AC} = 7i - 4j + 6i + 4j$

$= 13i + 0j$

$\vec{AC} = 6i + 4j$

$\vec{AB} = -4i + 6j$ just looking at the co-ordinates

$\vec{BA} = 4i - 6j$ $\qquad$ $\vec{BD} = \frac{1}{3}(\vec{BA}) = \frac{4}{3}i - 2j$

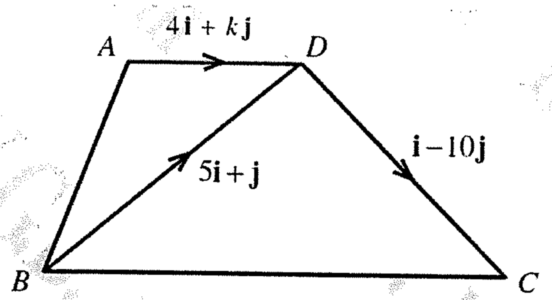
$\vec{OD} = \vec{OB} + \vec{BD} = 3i + 2j + \frac{4}{3}i - 2j = 5\frac{1}{3}i + 0j$

$= \frac{16}{3}i + 0j$

$\vec{OC} = \begin{pmatrix} 13 \\ 0 \end{pmatrix}$ $\quad$ $\vec{OD} : \begin{pmatrix} 16/3 \\ 0 \end{pmatrix}$ $\quad$ Scaler multiple

Ratio $\quad OC : \vec{OD}$

$39 : 16$



The figure above shows a trapezium $ABCD$, where AD is parallel to BC.

The following information is given for this trapezium.

$$\overrightarrow{BD} = 5\mathbf{i} + \mathbf{j}, \quad \overrightarrow{DC} = \mathbf{i} - 10\mathbf{j} \quad \text{and} \quad \overrightarrow{AD} = 4\mathbf{i} + k\mathbf{j},$$

where k is an integer.

a) Use vector algebra to show that $k = -6$.

b) Find the length of $\overrightarrow{AB}$.

c) Calculate the size of the angle ABD.

$\overrightarrow{BC} = \overrightarrow{BD} + \overrightarrow{DC}$

$= \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -10 \end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix} 6 \\ -9 \end{pmatrix}$ ✓

$\overrightarrow{AD} = \begin{pmatrix} 4 \\ k \end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix} 6 \\ -9 \end{pmatrix}$

These are parallel so scaler multiple

$4 \xrightarrow{\times 1.5} 6$

$k \xrightarrow{\times 1.5} -9$

Ans $k = -6$ ✓

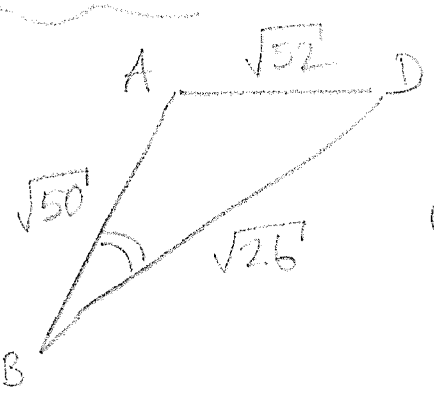
$\overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{DB}$

$= \begin{pmatrix} 4 \\ -6 \end{pmatrix} + \begin{pmatrix} -5 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ -7 \end{pmatrix}$

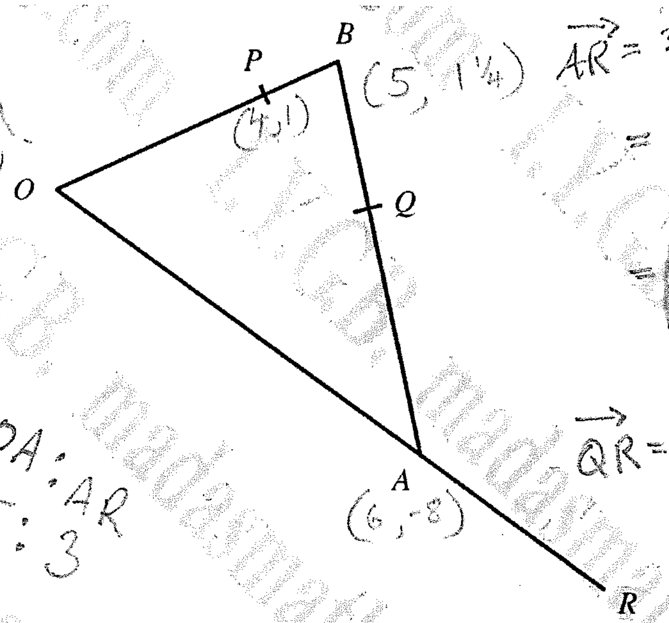
$|\overrightarrow{AB}| = \sqrt{1+49}$

$= \sqrt{50}$ ✓



use Cosine Rule

Really Tough One



$\overrightarrow{AR} = \frac{3}{5}\overrightarrow{OA}$

$= \frac{3}{5}\begin{pmatrix}6\\-8\end{pmatrix}$

$= \begin{pmatrix}\frac{18}{5}\\-\frac{24}{5}\end{pmatrix}$

$\overrightarrow{QR} = \overrightarrow{QA} + \overrightarrow{AR}$

$OA : AR$
$5 : 3$

The figure above shows a triangle OAB, where O is a fixed origin.

- The point A has coordinates $(6,-8)$.
- The point P, whose coordinates are $(4,1)$, lies on OB so that $OP : PB = 4:1$.
- The point Q lies on AB so that $AQ : QB = 3:2$
- The side OA is extended to the point R so that $OA : AR = 5:3$.

a) Use vector methods to determine the coordinates of Q.

b) Determine expressions, in terms of $\mathbf{i}$ and $\mathbf{j}$, for the vectors $\overrightarrow{PQ}$ and $\overrightarrow{QR}$.

c) Deduce, showing your reasoning, that P, Q and R are collinear and state the ratio of $PQ : QR$.

$\overrightarrow{OP} = \begin{pmatrix}4\\1\end{pmatrix}$ $\quad \overrightarrow{PB} = \begin{pmatrix}1\\ \frac{1}{4}\end{pmatrix}$ $\quad$ Point $B = (5, 1\tfrac{1}{4})$ ✓

$\overrightarrow{AB} = \begin{pmatrix}-1\\9\frac{1}{4}\end{pmatrix}$ $\quad AQ = \frac{3}{5}\overrightarrow{AB} = \frac{3}{5}\begin{pmatrix}-1\\9\frac{1}{4}\end{pmatrix} = \begin{pmatrix}-\frac{3}{5}\\ \frac{111}{20}\end{pmatrix}$

$\overrightarrow{PQ} = \overrightarrow{PB} + \overrightarrow{BQ}$

$\overrightarrow{PQ} = \begin{pmatrix}1\\ \frac{1}{4}\end{pmatrix} + \begin{pmatrix}\frac{2}{5}\\ -\frac{37}{10}\end{pmatrix}$

$= \begin{pmatrix}\frac{7}{5}\\ -\frac{69}{20}\end{pmatrix}$

$\overrightarrow{QR} = \overrightarrow{QA} + \overrightarrow{AR}$

$= \begin{pmatrix}\frac{3}{5}\\ -\frac{111}{20}\end{pmatrix} + \begin{pmatrix}\frac{18}{5}\\ -\frac{24}{5}\end{pmatrix}$

$= \begin{pmatrix}\frac{21}{5}\\ -\frac{207}{20}\end{pmatrix}$

Collinear because $3\overrightarrow{PQ} = \overrightarrow{QR}$

Relative to a fixed origin O on a horizontal plane, the points A and B have respective position vectors $3\mathbf{i}-2\mathbf{j}$ and $5\mathbf{i}+4\mathbf{j}$.

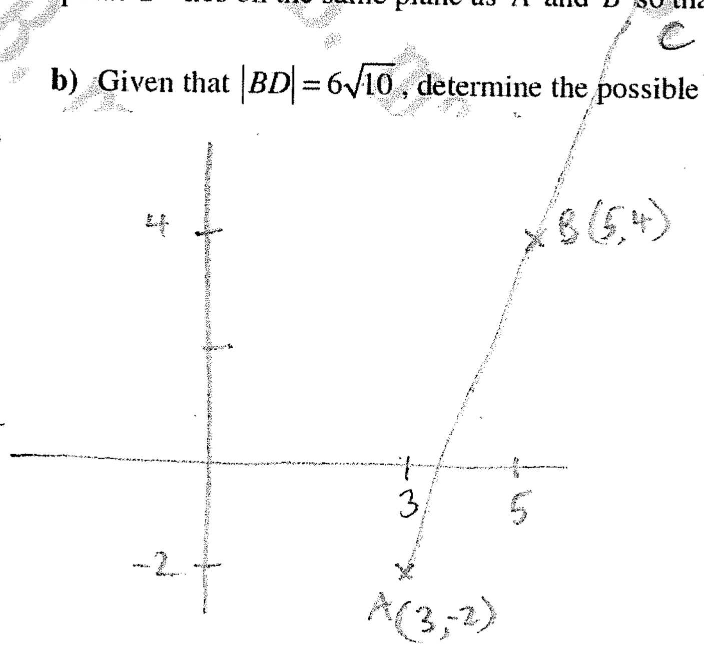
The point C lies on the same plane as A and B so that $\overline{AB} : \overline{BC} = 2:5$.

a) Find the position vector of C.

The point D lies on the same plane as A and B so that A, B and D are collinear.

b) Given that $|BD| = 6\sqrt{10}$, determine the possible position vectors of D.

(a)



$\overrightarrow{AB} : \overrightarrow{BC}$
$2 : 5$

C must be out past point B

$\overrightarrow{AB} = 2i + 6j$

$\overrightarrow{BC} = \frac{5}{2}(\overrightarrow{AB})$

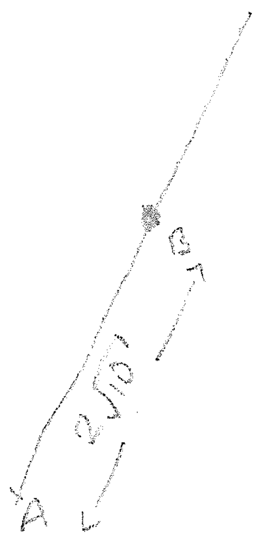
$\overrightarrow{BC} = 5i + 15j$

Point C is point $\underset{(5,4)}{B}$ then $\overrightarrow{BC}$

Ans $C = (10, 19)$

$|AB| = \sqrt{2^2 + 6^2}$
$= \sqrt{4 + 36}$
$= \sqrt{40}$
$= 2\sqrt{10}$

(b) A B D collinear



$6\sqrt{10} = 3 \times 2\sqrt{10} = 3 \times |AB|$

$\overrightarrow{AB} = 2i + 6j$

$\overrightarrow{BD} = 3 \times \overrightarrow{AB} = 6i + 18j$

$|\overrightarrow{BD}| = 6\sqrt{10}$

so $\overrightarrow{BD} = 6i + 18j$ or $-6i - 18j$

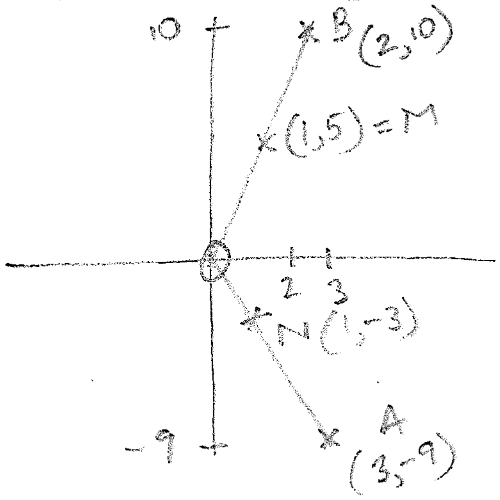
Possible $D = (11, 22)$ & $(-1, -14)$

Relative to a fixed origin O, the points A and B have position vectors $3\mathbf{i}-9\mathbf{j}$ and $2\mathbf{i}+10\mathbf{j}$, respectively.

The point M is the midpoint of OB and the point N lies on OA so that $\overline{OA} = 3\overline{ON}$.

The point P is the point of intersection of AM and BN.

Determine the ratio $\overline{NP} : \overline{PB}$.



$\overrightarrow{OB} = 2i + 10j$

$M = (1,5)$

$\overrightarrow{OA} = 3i - 9j$

$\overrightarrow{ON} = i - 3j$

$N = (1,-3)$

Line AM

$\overrightarrow{AM} = \begin{pmatrix}-2\\14\end{pmatrix}$

equation

$y = -7x + c$

Put in $(1,5)$

$5 = -7(1) + c$

$12 = c$

$y = -7x + 12$

Line BN

$\overrightarrow{NB} = \begin{pmatrix}1\\13\end{pmatrix}$

equation

$y = 13x + c$

Put in point $(2,10)$

$10 = 13(2) + c$

$10 = 26 + c$

$-16 = c$

$y = 13x - 16$

Point P is intersection of these 2 lines

$-7x + 12 = 13x - 16$

$28 = 20x$

$1.4 = x$

$y = 2.2$

$P = (1.4, 2.2)$

$\overrightarrow{NP} = 0.4i + 5.2j$

$\overrightarrow{PB} = 0.6i + 7.8j$

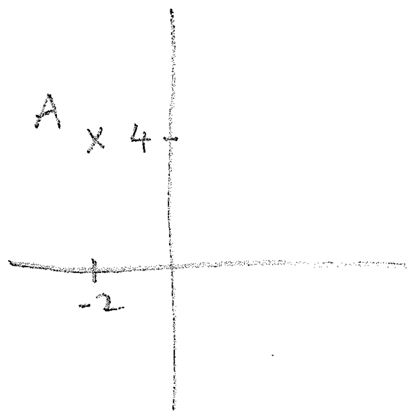
$\overrightarrow{NP} : \overrightarrow{PB} = 2 : 3$

Relative to a fixed origin O, the point A has coordinates $(-2,4)$.

The point B is such so that $\overrightarrow{BA} = 5\mathbf{i} - \mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are mutually perpendicular unit vectors lying on the same plane.

a) Determine the distance of B from O.

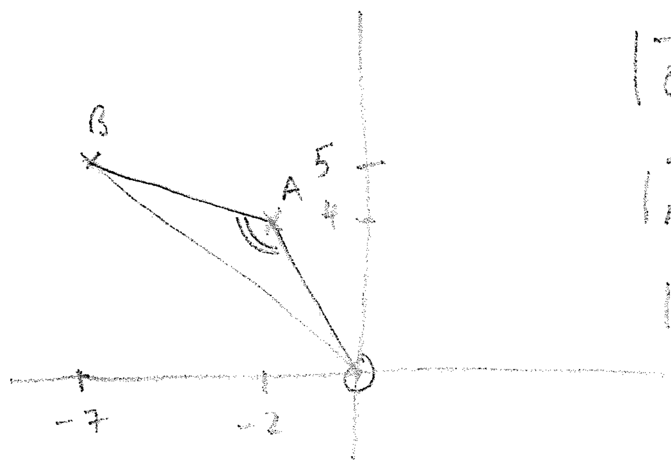
b) Calculate the angle OAB.



$A = (-2, 4)$

$\overrightarrow{AB} = -5i + j$

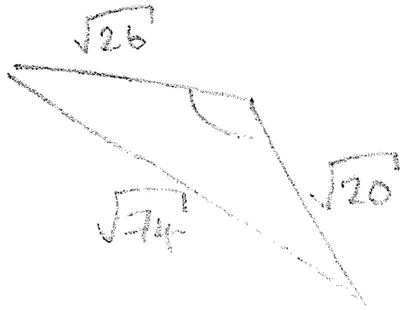
Point $B = -7i + 5j$



$|\overrightarrow{OA}| = \sqrt{4+16} = \sqrt{20}$

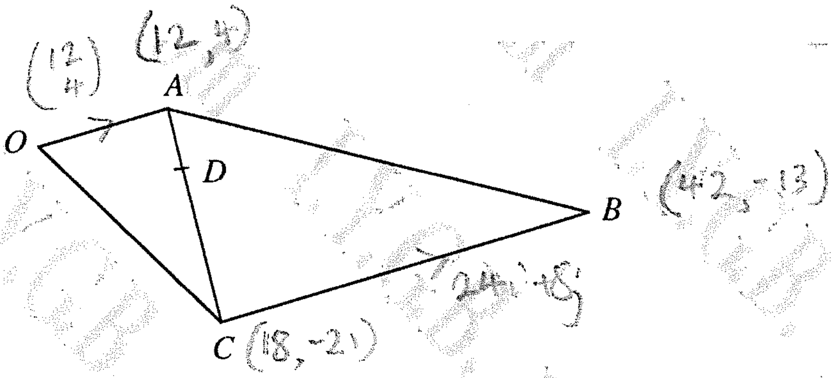
$|\overrightarrow{AB}| = \sqrt{25+1} = \sqrt{26}$

$|\overrightarrow{OB}| = \sqrt{49+25} = \sqrt{74}$



Cosine Rule

Easy from here!



The figure above shows a trapezium $OABC$, where O is a fixed origin.

The position vectors of A and C are $12\mathbf{i}+4\mathbf{j}$ and $18\mathbf{i}-21\mathbf{j}$, respectively.

CB is parallel to OA, so that $|\overrightarrow{CB}| = 2|\overrightarrow{OA}|$.

The point D lies on AC so that $AD : DC = 1:2$.

a) Find a simplified expression, in terms of $\mathbf{i}$ and $\mathbf{j}$, for the position vector of D.

b) Show that that O, D and B are collinear and state the ratio of $OD : DB$.

$\overrightarrow{OA} = \begin{pmatrix}12\\4\end{pmatrix}$

$\overrightarrow{AD} : \overrightarrow{DC}$
$1 : 2$

$\overrightarrow{AC} = 6i - 25j$

$\overrightarrow{AD} = \frac{1}{3}(\overrightarrow{AC}) = 2i - 8\frac{1}{3}j$

Point $D = ?$ $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \begin{pmatrix}12\\4\end{pmatrix} + \begin{pmatrix}2\\-8\frac{1}{3}\end{pmatrix} = \begin{pmatrix}14\\-4\frac{1}{3}\end{pmatrix}$

Point $D = (14, -4\frac{1}{3})$

$\overrightarrow{OD} = \begin{pmatrix}14\\-4\frac{1}{3}\end{pmatrix}$

Point $B = (42, -13)$

$\overrightarrow{OB} = \begin{pmatrix}42\\-13\end{pmatrix}$ $\overrightarrow{OB} = 3\times\overrightarrow{OD}$

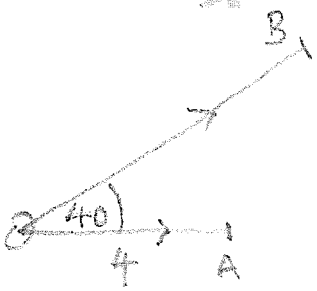
$OD : DB$
$1 : 2$

The points A, B and P lie on the x-y plane, where the point O is the origin.

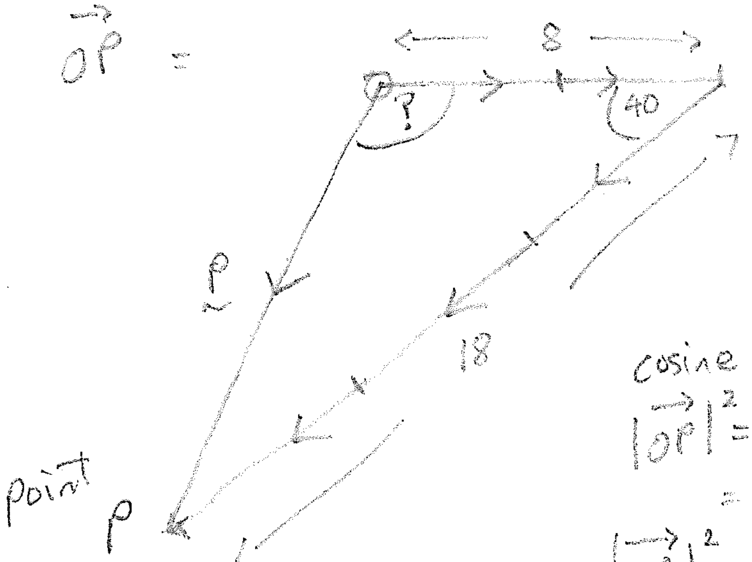
It is further given that

$$|OA| = 4, \quad |OB| = 6 \quad \text{and} \quad \measuredangle AOB = 40^\circ.$$

If $\overline{OP} = 2(\overline{OA}) - 3(\overline{OB})$ determine the distance of P from the origin and the angle between $\overline{OP}$ and $\overline{OA}$.



$\overrightarrow{OP} =$



$\overrightarrow{OP} = 2\overrightarrow{OA} - 3\overrightarrow{OB}$

Cosine Rule

$|\overrightarrow{OP}|^2 = 8^2 + 18^2 - 2 \times 8 \times 18 \times \cos 40$

$= 64 + 324 - 288 \cos 40$

$|\overrightarrow{OP}|^2 = 167.38$

$\overrightarrow{OP} = 12.938$

12.9

Sine Rule or Cosine Rule

$$\frac{\sin P\hat{O}A}{18} = \frac{\sin 40}{12.9}$$

$\sin P\hat{O}A = 0.894$

$P\hat{O}A = \cancel{63.4}$ or 116.6

Must be 116.6° because it must be biggest angle

The points $A(-1,4)$, $B(2,3)$ and $C(8,1)$ lie on the x-y plane, where O is the origin.

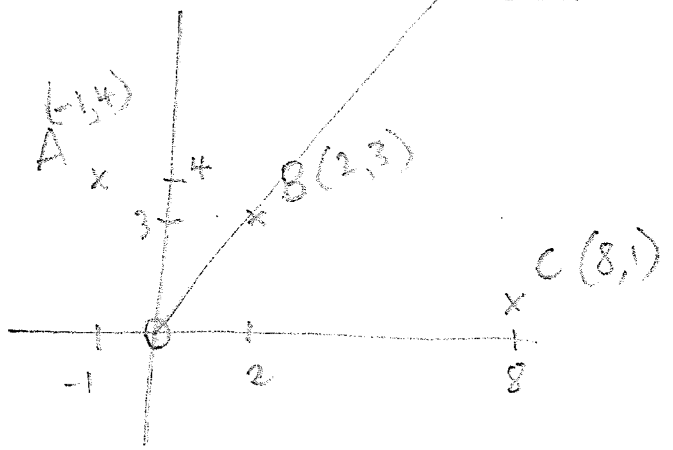
a) Show that A, B and C are collinear.

The point D lies on BC so that $\overline{BD} : \overline{BC} = 2:3$.

b) Find the coordinates of D.

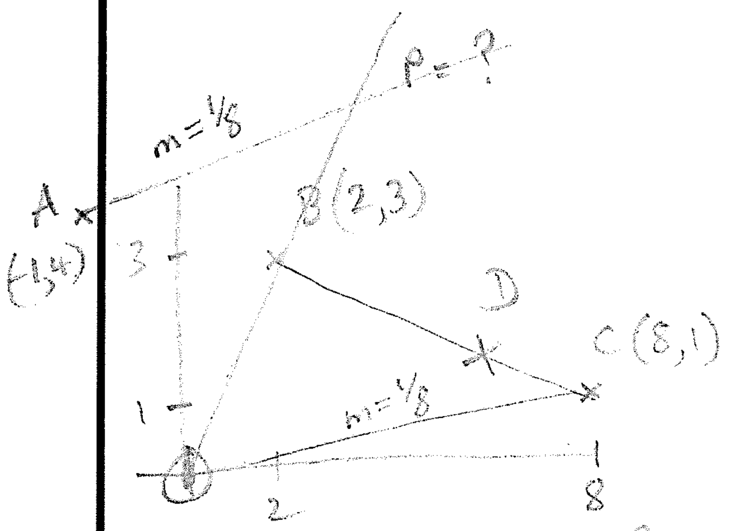
The straight line OB is extended to the point P, so that $\overline{AP}$ is parallel to $\overline{OC}$.

c) Determine the coordinates of P.



$\vec{AB} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ $\quad$ $\vec{AC} = \begin{pmatrix} 9 \\ -3 \end{pmatrix}$

Scalar multiple
$\vec{AC} = 3\vec{AB}$
so points are collinear



D is 2/3 allong BC

$BD : BC = 2:3$

$\vec{BC} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$

$\vec{BD} = \frac{2}{3}\begin{pmatrix} 6 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ -4/3 \end{pmatrix}$

D is $(6, 1\frac{2}{3})$

Line OB
$y = \frac{3}{2}x + 0$

Line AP
$y = \frac{1}{8}x + c$
Put in $(-1,4)$
$4 = \frac{1}{8}(-1) + c$
$4\frac{1}{8} = c$

$y = \frac{1}{8}x + 4\frac{1}{8}$

Solve these simultaneously