Arithmetic Progressions

Q1

A worker in a factory is given the task of sewing a total of 780 garments.
On the first day she completes 20 garments and plans to increase the number of garments she completes in a day by 2 every day.

(i) Find how many garments she will complete on the 16th day. [4]

(ii) Find how many days in total she will need to complete the task. [4]

$20, 22, 24, 26, \ldots$

Arithmetic progression

1^{st} term $= a = 20$ $\quad d = 2$

n^{th} term $= a + (n-1)d$

16^{th} term $= 20 + 15 \times 2 = 50$ Answer

(ii) How many days until total $= 780$

$a = 20 \quad d = 2$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$780 = \frac{n}{2}\{40 + (n-1)2\}$

$1560 = n\{38 + 2n\}$

$0 = 2n^2 + 38n - 1560$

$0 = 2(n + 19n - 780)$

$0 = 2(n - 20)(n + 39)$

$n = 20 \qquad n = -39$

impossible must be positive

Ans 20 days until 780 are made.

Q2

A married couple, Nicole and Brad, take out savings' investment plans.
Nicole plans to save £225 in the first year, £275 in the second year, £325 in the third year and so on increasing the annual amount saved by £50

$a = 225$ $d = 50$ A.P. $a,\ a+d,\ a+2d, \dots$

Using the fact that her planned savings form an arithmetic progression,

(i) find the amount that Nicole plans to save in the 10th year of her savings plan, [2]

n^{th} term $= a + (n-1)d$ 10^{th} year $= 225 + 9 \times 50 = 675$

(ii) find the **total** amount that Nicole plans to save over a 20 year period. [3]

$S = \frac{n}{2}\{2a + (n-1)d\} = \frac{20}{2}\{450 + 19 \times 50\} = 14000$

Brad plans to save £14 000 over a 20 year period.
He plans to save £P in the first year. His planned annual savings form an arithmetic progression with common difference £60

(iii) Find the value of P [3]

Brad wants Sum of 20 terms = 14000

$P,\ P+60,\ P+120,\ P+180, \dots$

$a = P$ $d = 60$

$S = \frac{n}{2}\{2a + (n-1)d\} = 14000$

$\frac{20}{2}\{2P + (19)60\} = 14000$

$2P + 1140 = 1400$

$2P = 1400 - 1140$

$2P = 260$

$P = 130$

Answer $P = £130$

check it

$130,\ 190,\ 250, \dots$ for 20 years

Q3

(i) Prove that the sum of the first n terms of an arithmetic series, with first term a and common difference d, is

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] \qquad [5]$$

You must be able to PROVE this formula

The sum of the first two terms of an arithmetic series is 2
The 41st term is 475

(ii) Show that the first term and the common difference are –5 and 12 respectively. [7]

(iii) Hence find the sum of the first 20 terms of this series. [2]

(i) $a \quad , a+d \quad , a+2d \quad \cdots \quad a+(n-1)d$

swap around the order

$$S_n = \boxed{a} + \boxed{a+d} + a+2d + \cdots + \boxed{a+(n-1)d}$$

$$S_n = \boxed{a+(n-1)d} + \boxed{a+(n-2)d} + \cdots + \boxed{a}$$

$$2S_n = \boxed{2a+(n-1)d} + \boxed{2a+(n-1)d} + \cdots + \boxed{2a+(n-1)d}$$

There are n of these

$$2S_n = n\left(2a+(n-1)d\right)$$

$$S_n = \frac{n}{2}\left(2a+(n-1)d\right)$$

(ii) $a + a+d = 2$
sum of first 2 terms

n^{th} term $= a+(n-1)d$

41^{st} term $= a+40d$

$$2a+d=2 \qquad\qquad a+40d = 475$$

$$d = 2-2a \qquad\qquad a+40(2-2a) = 475$$

$$a+80-80a = 475$$

$$-395 = 79a$$

$$d = 2-2(-5) \qquad\qquad -5 = a$$

$$d = 12$$

S of 20 terms

$$S_{20} = \frac{n}{2}\left\{2a+(n-1)d\right\} = \frac{20}{2}\left\{-10+19(12)\right\}$$

$$= 10\left\{-10+228\right\}$$

$$= 2180$$

Q4

A.P.

The first term of a Geometric Series is 4 and its common ratio is $\frac{1}{2}$
Find:

$a, ar, ar^2, ar^3, \ldots$

(i) the fifth term of the series; [2]

(ii) the sum to infinity of the series. [2]

Sneaky

$a =$ 1st term $\quad r =$ common ratio

$a = 4 \quad r = \frac{1}{2}$

n^{th} term $= ar^{n-1}$

5^{th} term $= 4 \times \left(\frac{1}{2}\right)^4$

Sum to infinity

$$S_\infty = \frac{a}{1-r}$$

Make sure you can prove this

$$S_\infty = \frac{4}{1-\frac{1}{2}} = 8$$

Q5

A sequence is defined recursively by

$$u_n = u_{n-1} - 3 \text{ where } u_1 = 5$$

(i) Find u_2 and u_3 $\quad u_1 = 5$ [2]

(ii) Using the fact that the terms of this sequence form an arithmetic progression, find its 600th term. [4]

$u_1 = 5$

$u_2 = u_1 - 3 = 5 - 3 = 2$

$u_3 = u_2 - 3 = 2 - 3 = -1$

Sequence $5, 2, -1, \dots$ A.P.

$a = 5 \quad d = -3$

n^{th} term $= a + (n-1)d$

600^{th} term $= 5 + (599)(-3)$

$= -1792$

Answer