Find the first 3 terms in ascending powers of x of the binomial expansion of $\left(2+\frac{x}{2}\right)^6$

Formula Page

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \dots$$

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots$$

You must choose the $(a+b)^n$ because you have $(2+\ )$

$$\left(2+\frac{x}{2}\right)^6 = 2^6 + \binom{6}{1}2^5\left(\frac{x}{2}\right) + \binom{6}{2}2^4\left(\frac{x}{2}\right)^2 + \dots$$

$$= 64 + 6\times 32\times\left(\frac{x}{2}\right) + 15\times 16\times\frac{x^2}{4} + \dots$$

$$= 64 + 96x + 60x^2 + \dots$$

The coefficient of x^2 in the binomial expansion of $(1 + \frac{2}{5}x)^n$, where n is a positive integer, is 1.6

a Find the value of n.

b Use your value of n to find the coefficient of x^4 in the expansion.

$(1+\frac{2}{5}x)^n$ because you don't know the n you must choose the 2nd formula in the formula booklet

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \frac{n(n-1)(n-2)}{1\times 2\times 3}x^3 + \dots$$

$$(1+\frac{2}{5}x)^n = 1 + n\left(\frac{2}{5}x\right) + \frac{n(n-1)}{1\times 2}\left(\frac{2}{5}x\right)^2 + \dots$$

$$= 1 + \frac{2n}{5}x + \frac{2(n)(n-1)}{25}x^2 + \dots$$

$$1.6 = \frac{2n(n-1)}{25}$$

$$40 = 2n(n-1)$$

$$20 = n(n-1)$$

$$0 = n^2 - n - 20$$

$$0 = (n-5)(n+4)$$

$n = 5$ Not negative

Ans $n = 5$

$\boxed{x^4 \text{ term}}$

$$\frac{n(n-1)(n-2)(n-3)}{1\times 2\times 3\times 4}\left(\frac{2}{5}x\right)^4$$

$$= \frac{5\times 4\times 3\times 2}{1\times 2\times 3\times 4} \times \frac{16x^4}{625} = \frac{16}{125}x^4$$

Ans $\frac{16}{125}$

$$f(x) = (x + 3)(x - 1)^2$$

(a) Sketch the curve $y = f(x)$, showing the points of intersection with the coordinate axis.

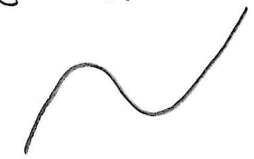
(b) Find the equation of $y = f(x + 2)$ in the form $y = (x + a)(x + b)^2$

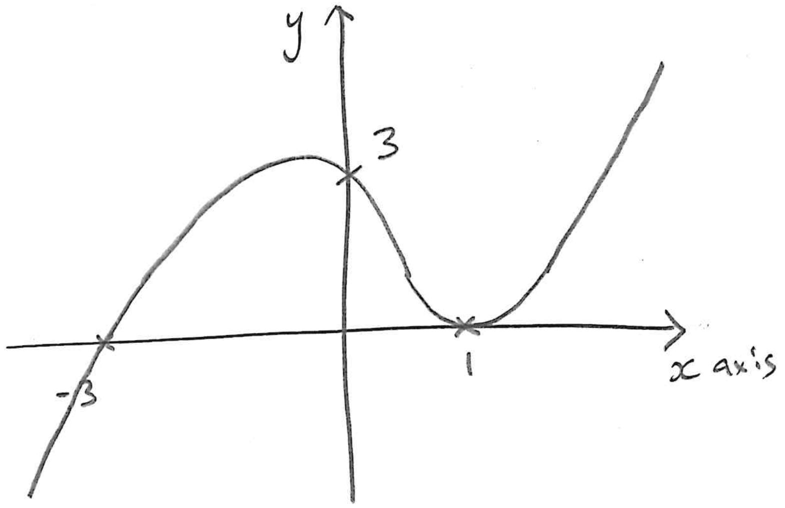
$y = (x+3)(x-1)^2$

root at $(-3, 0)$

repeated root at $(1,0)$

x^3 curve looks like





cuts y axis when $x = 0$

$f(0) = 3(-1)^2$

$= 3 \times 1$

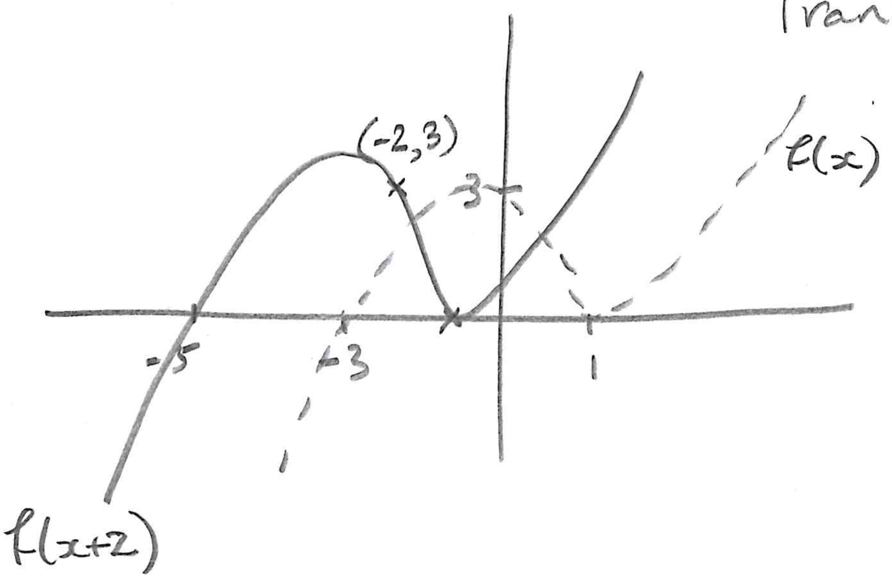
$= 3$

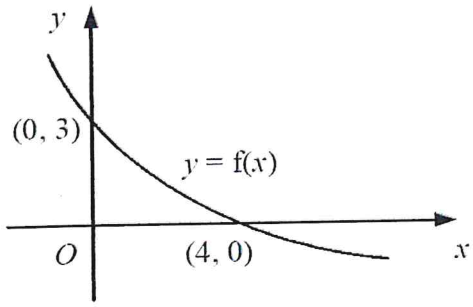
$f(x+2) = ?$

$f(x+2) = (x+2+3)(x+2-1)^2$

$f(x+2) = (x+5)(x+1)^2$

Beyoncé Transformation



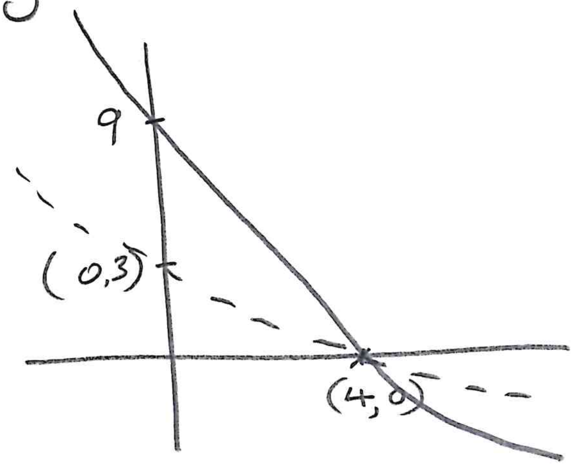


The diagram shows the curve with equation $y = f(x)$ which crosses the coordinate axes at the points (0, 3) and (4, 0).

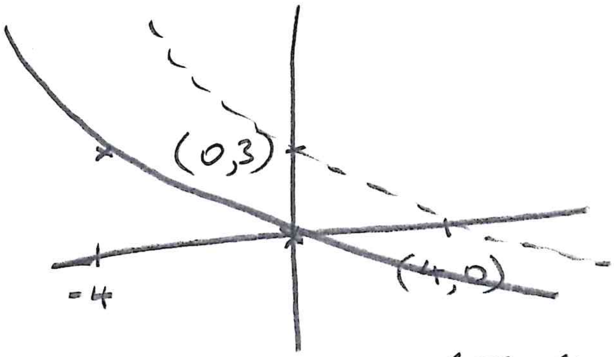
Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of

a $y = 3f(x)$ **b** $y = f(x + 4)$ **c** $y = -f(x)$ **d** $y = f(\frac{1}{2}x)$

(a) $3f(x)$
outside the bracket means it affects the y co-ordinate

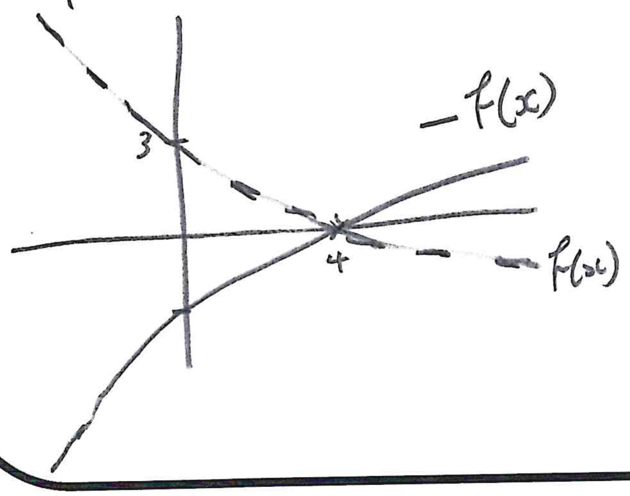


(b) $y = f(x+4)$
Beyoncé Transformation
$(x+4)$ inside the bracket so this influences the x co-ordinate

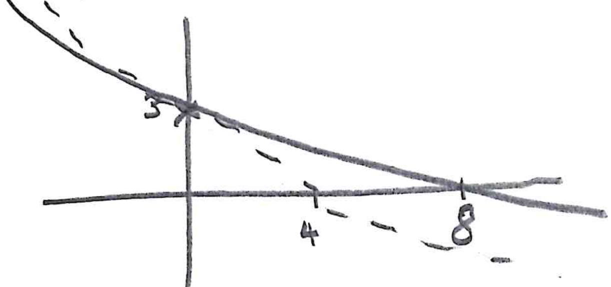


to the left

(c) $y = -f(x)$
flips over the x axis



(d) $y = f(\frac{1}{2}x)$
inside the bracket influences the x numbers



$\times 2$

(a) Find the first 3 terms in ascending powers of x of the binomial expansion of $\left(2-\frac{x}{8}\right)^7$

$f(x) = (ax+b)\left(2-\frac{x}{8}\right)^7$ where a and b are constants

use $(a+b)^7$

Given that the first two terms, in ascending powers of x, in the series expansion of $f(x)$ are 384 and $-104x$

(b) Find the values of a and b

$$\left(2-\frac{x}{8}\right)^7 = 2^7 + \binom{7}{1}2^6\left(\frac{-x}{8}\right) + \binom{7}{2}2^5\left(\frac{-x}{8}\right)^2 + \cdots$$

$$= 128 + 7\times 64\left(\frac{-x}{8}\right) + 21\times 32\times\left(\frac{x^2}{64}\right) + \cdots$$

$$= 128 - 56x + \frac{21}{2}x^2 + \cdots$$

$$(ax+b)\left(2-\frac{x}{8}\right)^7$$

not required

$$(ax+b)\left(128 - 56x \cancel{+ \frac{21}{2}x^2} + \cdots\right)$$

$$(ax+b)(128 - 56x + \cdots)$$

$$128ax - \cancel{56ax^2} + 128b - 56bx + \cdots$$

not required

$$128b + (128a - 56b)x + \cdots$$

$$384 = 128b$$

$$3 = b$$

$$-104 = 128a - 56b$$

$$-104 = 128a - 56(3)$$

$$-104 = 128a - 168$$

$$64 = 128a$$

$$\frac{1}{2} = a$$

Ans $a = \frac{1}{2}$ $b = 3$

(a) Find the first four terms, in ascending powers of x, of the binomial expansion $(2 + kx)^6$

Given that the coefficient of the x^3 term in the expansion is -20

(b) Find the value of k

$(2+kx)^6$ Because you know 6 then you should use first formula

$$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \dots$$

$$(2+kx)^6 = 2^6 + {}^6C_1 2^5 (kx) + {}^6C_2 2^4 (kx)^2 + {}^6C_3 2^3 (kx)^3 + \dots$$

$${}^6C_3 2^3 (kx)^3$$

$$20 \times 8 \times k^3 x^3$$

But $160k^3x^3 = -20x^3$

$$160k^3 = -20$$

$$k^3 = -1/8$$

$$k = \sqrt[3]{-1/8}$$

$$k = -1/2$$

$$f(x) = x^2 + 4x + 5$$

(a) Express $f(x)$ in the form $(x + a)^2 + b$, and state the coordinates of the minimum point of $y = f(x)$. **(3)**

(b) Sketch the graph of $y = f(x)$ showing the coordinates of intersection with the coordinate axis. **(3)**

(c) Find the minimum points of these curves

(i) $y = 2f(x)$ **(2)**

(ii) $y = f(2x)$ **(2)**

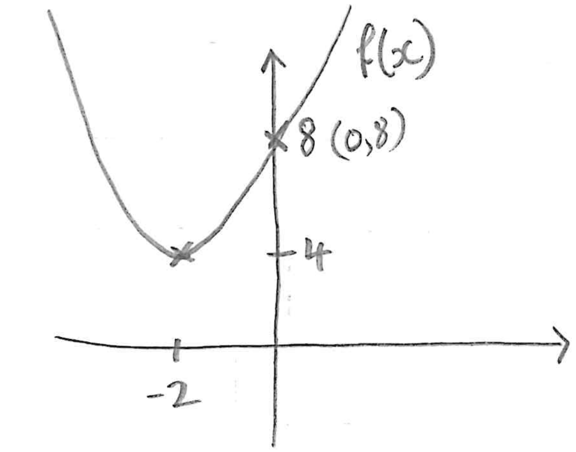
$f(x) = x^2 + 4x + 5$

$f(x) = (x+2)^2 + 5 - 1 = (x+2)^2 + 4$

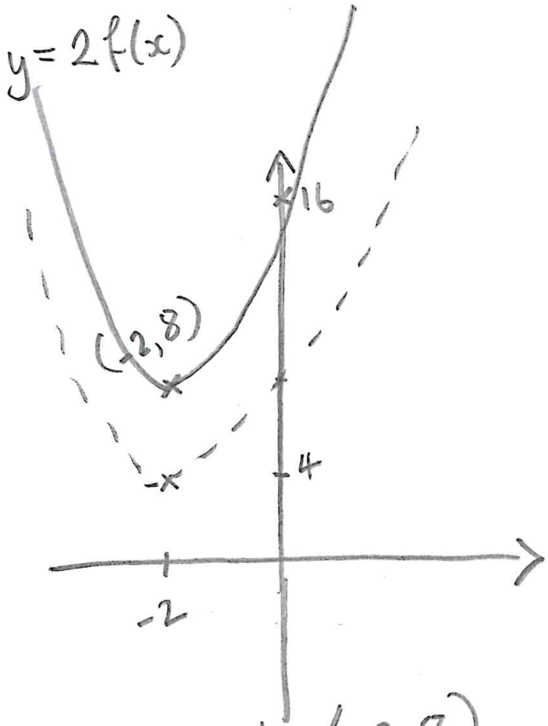
Minimum $(-2, 4)$

because $(\;)^2 + 4$

zero when $x = -2$



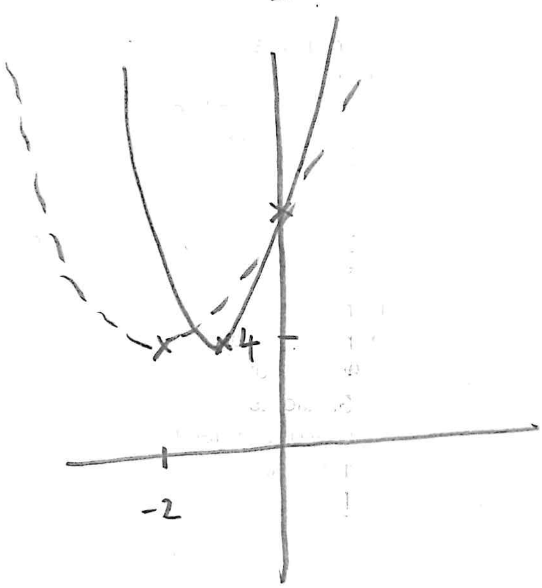
$y = 2f(x)$



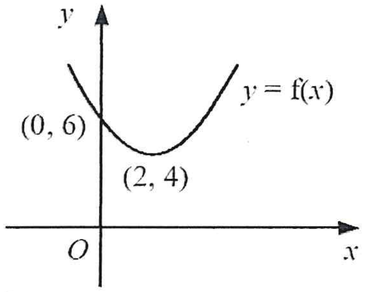
min point $(-2, 8)$

$y = f(2x)$

$\longrightarrow$ $\frac{1}{2}$ $\longleftarrow$



min point $(-1, 4)$



The diagram shows the curve with equation $y = f(x)$ which has a turning point at (2, 4) and crosses the y-axis at the point (0, 6).

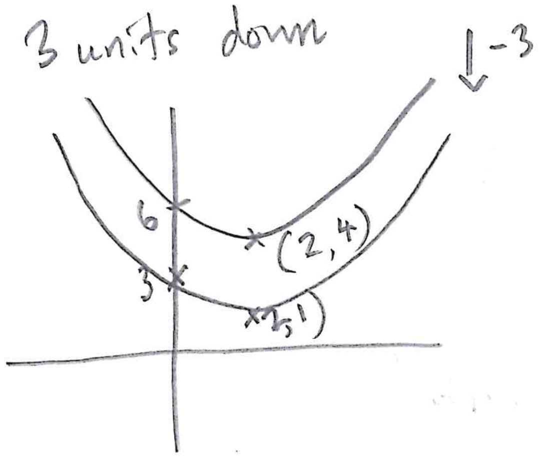
Showing the coordinates of the turning point and of any points of intersection with the axes, sketch on separate diagrams the graphs of

a $y = f(x) - 3$ **b** $y = f(x + 2)$ **c** $y = f(2x)$ **d** $y = \frac{1}{2}f(x)$

(a) y = curve $- 3$

so curve translates

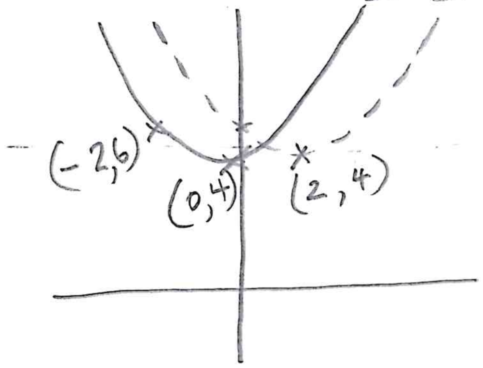
3 units down



(b) $f(x+2)$

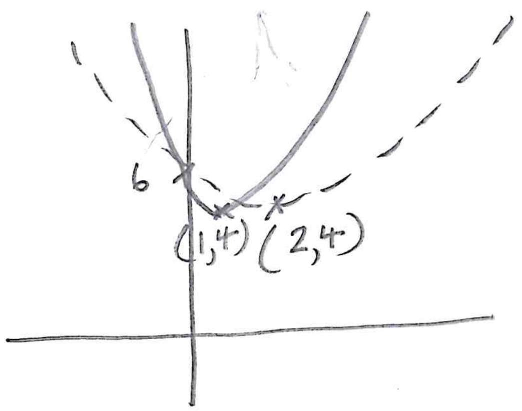
Beyoncé

2 units to the left



(c) $y = f(2x)$

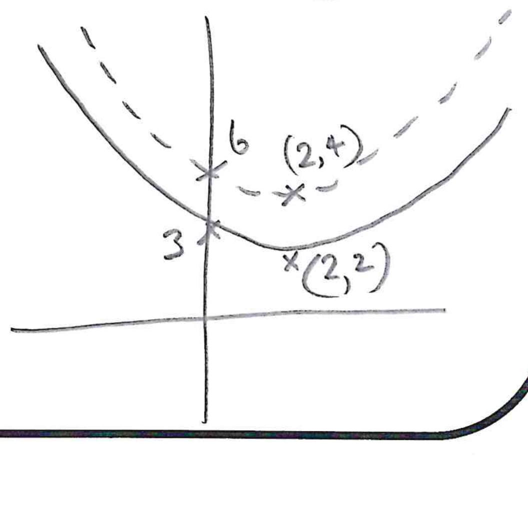
squashed in $\rightarrow$ $\leftarrow$



$y = \frac{1}{2}f(x)$

to do with y co-ords

½ of y co-ords



The coefficient of x^2 in the expansion of $(1 + ax)^4$ in ascending powers of x is 24, where a is a constant and $a < 0$. Find

a the value of a,

b the value of the coefficient of x^3 in the expansion.

$$(1+ax)^4 = 1 + 4(ax) + \underbrace{\frac{4\times3}{1\times2}(ax)^2}_{24x^2} + \frac{4\times3\times2}{1\times2\times3}(ax)^3 + \dots$$

so $\frac{4\times3}{1\times2}(ax)^2 = 24x^2$

$6a^2x^2 = 24x^2$

$a^2 = 4$

$a = -2$ can't be $+2$

$$(1-2x)^4 = \dots + \frac{4\times3\times2}{1\times2\times3}(-2x)^3 + \dots$$

$4(-2x)^3$

$4(-8x^3)$

$-32x^3$

Ans -32 is coefficient

In the binomial expansion of $(1 + px)^q$, where p and q are constants and q is a positive integer, the coefficient of x is -12 and the coefficient of x^2 is 60.

Find

a the value of p and the value of q,

b the value of the coefficient of x^3 in the expansion.

$(1+px)^q$

use 2nd Bino Expansion from Formula Booket

ALWAYS use 2nd expansion when you don't know the index number.

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \frac{n(n-1)(n-2)}{1\times 2\times 3}x^3 + \dots$$

$$(1+px)^q = 1 + q(px) + \frac{q(q-1)}{1\times 2}(px)^2 + \frac{q(q-1)(q-2)}{1\times 2\times 3}(px)^3 + \dots$$

$$= 1 - 12x + 60x^2 + \dots$$

$pq = -12$

$p = \frac{-12}{q}$

$\frac{q(q-1)}{2}p^2 = 60$

$q(q-1)p^2 = 120$

$q(q-1)\left(\frac{-12}{q}\right)^2 = 120$

$144q(q-1) = 120q^2$

$24q^2 - 144q = 0$

$24q(q-6) = 0$

$q = 6$

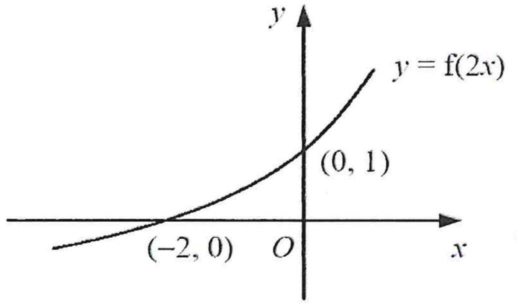
means $p = -2$

(b) x^3 term

$\frac{q(q-1)(q-2)}{1\times 2\times 3}(px)^3$

$\frac{6\times 5\times 4}{1\times 2\times 3}(-2x)^3$

$-160x^3$



The diagram shows the curve with equation $y = f(2x)$ which crosses the coordinate axes at the points (−2, 0) and (0, 1).

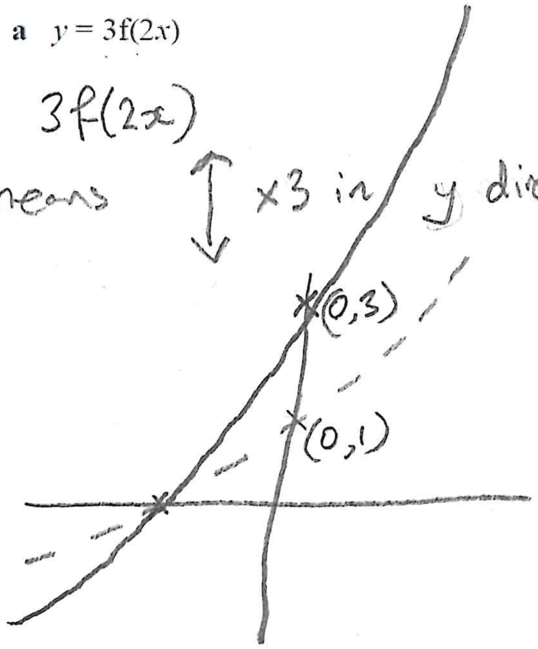
Showing the coordinates of any points of intersection with the coordinate axes, sketch on separate diagrams the curves

a $y = 3f(2x)$

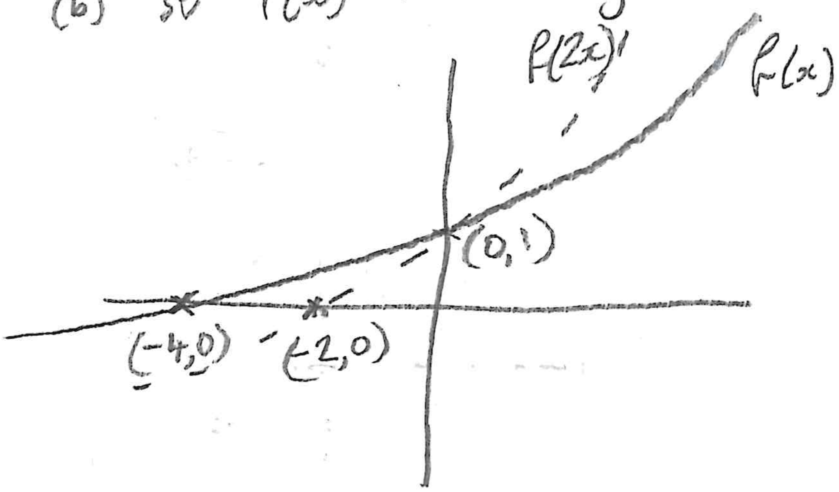
b $y = f(x)$

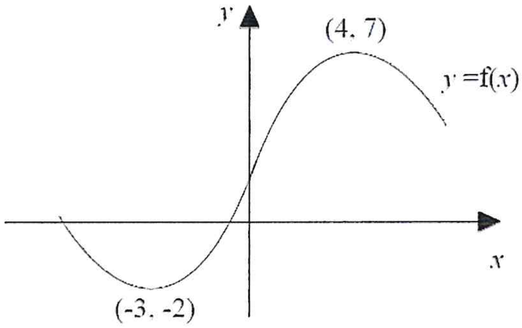
$3f(2x)$

means ↕ ×3 in y direction



(b) so $f(x)$ means you are finding $f(\frac{1}{2}2x)$





The sketch shows the graph of y = f(x). The curve has a minimum at (-3,-2) and a maximum at (4,7).

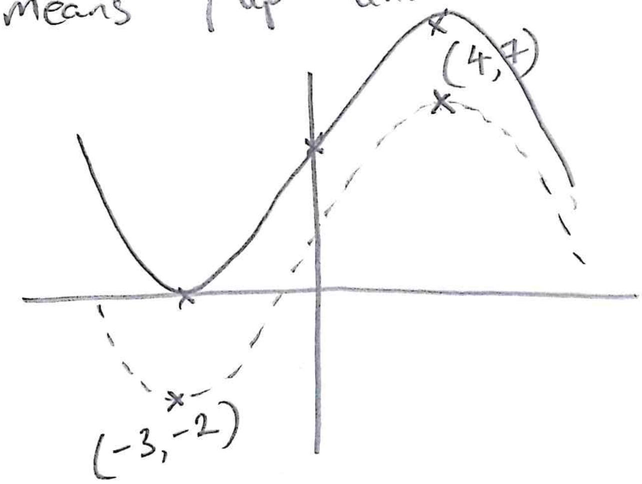
Showing the coordinates of the points of intersection with the coordinate axis, sketch on separate diagrams the curves

(i) $y = f(x) + 2$ **(2)**

(ii) $y = -f(x)$ **(2)**

(i) $f(x) + 2$

means ↑ up 2 units



(ii) $-f(x)$ means everything that was
+ive is now −ive
−ive is now +ive
so <u>reflect</u> everything in x axis