$$g(x) = (2 + ax)^8 \qquad \text{where } a \text{ is a constant}$$

Given that one of the terms in the binomial expansion of $g(x)$ is $3402x^5$

(a) find the value of a.

(4)

Using this value of a,

(b) find the constant term in the expansion of

$$\left(1 + \frac{1}{x^4}\right)(2 + ax)^8$$

(3)

$${}^8C_5(2)^3(ax)^5 = 3402x^5$$

$$448a^5x^5 = 3402x^5$$

$$448a^5 = 3402$$

$$a^5 = \frac{243}{32}$$

$$a = \frac{3}{2}$$

b) $\left(1 + \frac{1}{x^4}\right)\left(2 + \frac{3}{2}x\right)^8$

$$\left(1 + \frac{1}{x^4}\right)\left(2^8 + {}^8C_1(2)^7\left(\tfrac{3}{2}x\right) + {}^8C_2(2)^6\left(\tfrac{3}{2}x\right)^2 + {}^8C_3(2)^5\left(\tfrac{3}{2}x\right)^3 + {}^8C_4(2)^4\left(\tfrac{3}{2}x\right)^4 \ldots\right)$$

$$\left(1 + \frac{1}{x^4}\right)\left(2^8 + \ldots + {}^8C_4(2)^4\left(\tfrac{3}{2}x\right)^4 + \ldots\right)$$

Constant terms: $2^8 + \frac{1}{\cancel{x^4}}\left({}^8C_4(2)^4\left(\tfrac{3}{2}\right)^4\cancel{x^4}\right)$

$$= 5926$$

(a) Find the first 4 terms, in ascending powers of x, in the binomial expansion of

$$(1 + kx)^{10}$$

where k is a non-zero constant. Write each coefficient as simply as possible.

(3)

Given that in the expansion of $(1 + kx)^{10}$ the coefficient x^3 is 3 times the coefficient of x,

(b) find the possible values of k.

(3)

a/ 1 10 45 120

$1(1)^{10} + 10(1)^9(kx) + 45(1)^8(kx)^2 + 120(1)^7(kx)^3$

$1 + 10kx + 45k^2x^2 + 120k^3x^3$

b/ $120k^3 = 3(10k)$

$120k^3 = 30k$

$4k^3 = k$

$4k^3 - k = 0$

$k(4k^2 - 1) = 0$ k is not zero

$4k^2 - 1 = 0$

$4k^2 = 1$

$k^2 = \frac{1}{4}$

$k = \pm\sqrt{\frac{1}{4}}$

$\underline{\underline{k = \pm\frac{1}{2}}}$

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 + \frac{3x}{4}\right)^6$$

giving each term in its simplest form.

(4)

(b) Explain how you could use your expansion to estimate the value of 1.925^6
You do not need to perform the calculation.

(1)

$1 \quad 6 \quad 15$

a/ $(2)^6 + 6(2)^5\left(\frac{3x}{4}\right) + 15(2)^4\left(\frac{3x}{4}\right)^2$

$\underline{64 + 144x + 135x^2}$

b/ $2 + \frac{3x}{4} = 1.925$

$\frac{3}{4}x = \frac{-3}{40}$

$\underline{x = -0.1}$

We could substitute $x = -0.1$ into the expansion

(a) Use the binomial theorem to find all the terms of the expansion of

$$(2 + 3x)^4$$

Give each term in its simplest form.

(4)

(b) Write down the expansion of

$$(2 - 3x)^4$$

in ascending powers of x, giving each term in its simplest form.

(1)

1 4 6 4 1

a) $(2)^4 + 4(2)^3(3x) + 6(2)^2(3x)^2 + 4(2)(3x)^3 + (3x)^4$

$16 + 96x + 216x^2 + 216x^3 + 81x^4$

b) $16 - 96x + 216x^2 - 216x^3 + 81x^4$

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{x}{16}\right)^9$$

giving each term in its simplest form.

(4)

$$f(x) = (a + bx)\left(2 - \frac{x}{16}\right)^9, \text{ where } a \text{ and } b \text{ are constants}$$

Given that the first two terms, in ascending powers of x, in the series expansion of f(x) are 128 and 36x,

(b) find the value of a,

(2)

(c) find the value of b.

(2)

$1 \quad 9 \quad 36$

a) $(2)^9 + 9(2)^8\left(\frac{-x}{16}\right) + 36(2)^7\left(\frac{-x}{16}\right)^2$

$512 - 144x + 18x^2$

b) $(512 - 144x + 18x^2)(a + bx)$

$512a + 512bx - 144ax \ldots$

$512a = 128$

$a = \frac{1}{4}$

$512b - 144a = \cancel{144}\ 36$ [x terms]

$512b - 144\left(\frac{1}{4}\right) = \cancel{144}\ 36$

$512b - 36 = \cancel{144}\ \cancel{72}\ 36$

$512b = \cancel{108}\ 72$

$b = \frac{9}{64}$

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of $\left(2-\frac{x}{2}\right)^7$, giving each term in its simplest form.

(4)

(b) Explain how you would use your expansion to give an estimate for the value of 1.995^7

(1)

a/ $1(2)^7 + 7(2)^6\left(-\frac{x}{2}\right) + 21(2)^5\left(\frac{-x}{2}\right)^2$

$128 - 224x + 168x^2$

b/ $2 - \frac{x}{2} = 1.995$

$2 = 1.995 + \frac{x}{2}$

$0.005 = \frac{x}{2}$

$\underline{x = 0.01}$

Substitute $x = 0.01$ into the expansion

Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(3 - \frac{1}{3}x\right)^5$$

giving each term in its simplest form.

(4)

$1, \ 5, \ 10, \ 10$

$1(3)^5 + 5(3)^4\left(-\frac{1}{3}x\right) + 10(3)^3\left(-\frac{1}{3}x\right)^2 + 10(3)^2\left(-\frac{1}{3}x\right)^3$

$243 - 135x + 30x^2 - \frac{10}{3}x^3$

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 9x)^4$$

giving each term in its simplest form.

(4)

$$f(x) = (1 + kx)(2 - 9x)^4, \text{ where } k \text{ is a constant}$$

The expansion, in ascending powers of x, of $f(x)$ up to and including the term in x^2 is

$$A - 232x + Bx^2$$

where A and B are constants.

(b) Write down the value of A.

(1)

(c) Find the value of k.

(2)

(d) Hence find the value of B.

4^{th} line $\quad nC0 \quad nC1 \quad nC2$

$1 \quad 4 \quad 6$

$1(2)^4 + 4(2)^3(-9x) + 6(2)^2(-9x)^2$

$16 - 288x + 1944x^2$

b/ $(1+kx)(16 - 288x + 1944x^2)$

$16 - 288x + 1944x^2 + 16kx - 288kx^2 + 1944kx^3$

$16 - 288x + 16kx + 1944x^2 - 288kx^2$

$A = 16$

Equating x terms

c/ $-288 + 16k = -232$

$16k = 56$

$k = \frac{7}{2}$

d/ $1944 - 288\left(\frac{7}{2}\right) = B$

$936 = B$

Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{x}{4}\right)^{10}$$

giving each term in its simplest form.

(4)

1 10 45

$1(2)^{10} + 10(2)^{9}\left(-\frac{x}{4}\right) + 45(2)^{8}\left(-\frac{x}{4}\right)^{2}$

$1024 + -1280x + 720x^{2}$

$1024 - 1280x + 720x^{2}$

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2-3x)^6$$

giving each term in its simplest form.

(4)

(b) Hence, or otherwise, find the first 3 terms, in ascending powers of x, of the expansion of

$$\left(1+\frac{x}{2}\right)(2-3x)^6$$

(3)

3a) $(2-3x)^6$

1 6 15

$1(2)^6 + 6(2)^5(-3x) + 15(2)^4(-3x)^2$

$\underline{64 - 576x + 2160x^2}$

3b) $\left(1+\frac{x}{2}\right)\left(64-576x+2160x^2\right)$

$64 + 32x - 576x + 2160x^2 - 288x^2$

$\underline{64 - 544x + 1872x^2}$

Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{1}{2}x\right)^8$$

giving each term in its simplest form.

(4)

1 8 28 56

$1(2)^8 + 8(2)^7\left(-\frac{1}{2}x\right) + 28(2)^6\left(-\frac{1}{2}x\right)^2 + 56(2)^5\left(-\frac{1}{2}x\right)^3$

$256 - 512x + 448x^2 - 224x^3$

Find the first 3 terms, in ascending powers of x, in the binomial expansion of

$$(2-5x)^6$$

Give each term in its simplest form.

(4)

$1 \qquad 6 \qquad 15$

$1(2)^6 + 6(2)^5(-5x) + 15(2)^4(-5x)^2$

$64 \; \cancel{+} - 960x + 6000x^2$