Solve the equation

$$3^{x+1} = \frac{27^x}{9} \qquad [5]$$

3, 27 & 9 are all connected to 3

$$3^{x+1} = \frac{(3^3)^x}{3^2}$$

$$3^{x+1} = \frac{3^{3x}}{3^2}$$

$$3^2 \times 3^{x+1} = 3^{3x}$$

$$3^{x+3} = 3^{3x}$$

$$x+3 = 3x$$

$$3 = 2x$$

$$\frac{3}{2} = x$$

Find x given that

$$\frac{5^x}{25^{x-1}} = \sqrt{5}$$ [6]

5 & 25 are connected to 5

$$\frac{5^x}{(5^2)^{x-1}} = 5^{1/2}$$

$$\frac{5^x}{5^{2x-2}} = 5^{1/2}$$

$$5^x = 5^{1/2} \times 5^{2x-2}$$

$$5^x = 5^{2x - 3/2}$$

Now looking only at indices

$$x = 2x - 3/2$$

$$3/2 = x$$

Ans $x = 3/2$ ✓

Solve the equation

$$27x^{-\frac{1}{2}} = 125x$$

[5]

$$\frac{27}{x^{1/2}} = 125x$$

$$27 = 125x \times x^{1/2}$$

$$\frac{27}{125} = x^{3/2}$$

$$\sqrt[3]{\frac{27}{125}} = x^{1/2}$$

$$\frac{3}{5} = x^{1/2}$$

$$\frac{9}{25} = x$$

Check again

Solve the equation

$$\frac{16^x}{2^{x-1}} = 2^{\frac{1}{2}}$$

[6]

2 & 16 are connected

$16 = 2^4$

$$\frac{16^x}{2^{x-1}} = 2^{1/2}$$

$$\frac{(2^4)^x}{2^{x-1}} = 2^{1/2}$$

$$2^{4x} = 2^{1/2} \times 2^{x-1}$$

$$2^{4x} = 2^{x-1/2}$$

Looking only at indices

$$4x = x - 1/2$$

$$3x = -1/2$$

$$x = -1/6$$

Check it out! ✓

Solve the simultaneous equations

$$4x - 3y = 11$$

and $27^x \times 9^{y+3} = 3\sqrt{3}$ [9]

all connect to 3

$27^x \times 9^{y+3} = 3 \times 3^{1/2}$

$(3^3)^x \times (3^2)^{y+3} = 3^{3/2}$

$3^{3x} \times 3^{2y+6} = 3^{3/2}$

$3^{3x+2y+6} = 3^{3/2}$

Now $3x + 2y + 6 = 3/2$ & $4x - 3y = 11$

$3x + 2y = -9/2$

$\boxed{6x + 4y = -9}$ & $\boxed{4x - 3y = 11}$

Solve simultaneously

$x = 1/2 \quad y = -3$

You should check

Solve the equation

$$\frac{81^{3-x}}{27^{2x+1}} = 3$$

[6]

81, 27 & 3 are all connected to 3

3^4, 3^3 & 3

$$\frac{(3^4)^{3-x}}{(3^3)^{2x+1}} = 3$$

$$3^{12-4x} = 3^1 \times 3^{6x+3}$$

$$3^{12-4x} = 3^{6x+4}$$

Now

$$12 - 4x = 6x + 4$$

$$12 - 4 = 6x + 4x$$

$$8 = 10x$$

$$0.8 = x$$

check it out!

$$\frac{81^{2.2}}{27^{2.6}} = 3$$ ✓ on the new calculator!

Solve

$$\frac{27^x}{3^{x-1}} = 3\sqrt{3} \qquad [6]$$

Big Hint:
This is INDICES because $27 = 3^3$

$$\frac{27^x}{3^{x-1}} = 3 \times 3^{1/2}$$

$$(3^3)^x = 3^{3/2} \times 3^{x-1}$$

$$3^{3x} = 3^{x+1/2}$$

Now

$$3x = x + 1/2$$

$$2x = 1/2$$

$$x = 1/4$$

We should check these!

$x = 1/4$

$$\frac{27^{1/4}}{3^{1/4-1}} \quad \checkmark \quad = 3\sqrt{3}^1$$

Solve the equation

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

Big Hint this is an indices question

4 is 2^2 so we can write as $2^?$

$$4^{3x-2} = 2^{-1} \times 2^{-1/2}$$

$$(2^2)^{3x-2} = 2^{-3/2}$$

$$2^{6x-4} = 2^{-3/2}$$

so now

$$6x - 4 = -3/2$$

$$6x = -3/2 + 4$$

$$6x = 5/2$$

$$x = 5/12$$

We should really check this!

4^{3x-2} $\qquad$ $2^{-3/2}$

$4^{15/12 - 2}$

$4^{-3/4}$

0.353355 $\qquad$ 0.353355 ✓

Answer

$x = 5/12$

Solve the equation

$$\left(\tfrac{1}{2}\right)^{1-x} = \left(\tfrac{1}{8}\right)^{2x}$$

Numbers 2 & 8 are connected

$2^3 = 8$

Put all into $2^?$

$$\left(\tfrac{1}{2}\right)^{1-x} = \left(\tfrac{1}{8}\right)^{2x}$$

$$\left(2^{-1}\right)^{1-x} = \left(2^{-3}\right)^{2x}$$

$$2^{x-1} = 2^{-6x}$$

so

$$x - 1 = -6x$$

$$7x = 1$$

$$x = 1/7$$

Looks a bit strange so let's check it

LHS $\left(\tfrac{1}{2}\right)^{1-1/7}$

$\left(\tfrac{1}{2}\right)^{6/7}$

on calculator

0.552044

RHS $\left(\tfrac{1}{8}\right)^{2\times 1/7}$

$\left(\tfrac{1}{8}\right)^{2/7}$

0.552044

Amazingly true.

So $x = 1/7$

a Given that $y = 2^x$, express each of the following in terms of y.

i 2^{x+2} **ii** 4^x

b Hence, or otherwise, find the value of x for which

$$4^x - 2^{x+2} = 0.$$

Quadratic looking!

$2^{x+2} = 2^x \times 2^2 = 4(2^x) = 4y$

$4^x = (2^2)^x = (2^x)^2 = y^2$

so now solve

$4^x - 2^{x+2} = 0$

$y^2 - 4y = 0$

$y(y-4) = 0$

$y = 0$ $y = 4$

$2^x = 0$ $2^x = 4$

$x =$ not possible $x = 2$ only one answer

question says "find the value"

Ans $x = 2$

Solve the simultaneous equations

$$4^{2x} = 2^{y-1}$$

$$9^{4x} = 3^{y+1}$$

$4^{2x} = 2^{y-1}$

Big Hint

$4 = 2^2$

so 2 & 4 are connected

$(2^2)^{2x} = 2^{y-1}$

$2^{4x} = 2^{y-1}$

$\boxed{4x = y-1}$

$9^{4x} = 3^{y+1}$

Big Hint 3 & 9 are connected

$(3^2)^{4x} = 3^{y+1}$

$3^{8x} = 3^{y+1}$

$\boxed{8x = y+1}$

solve these simultaneously

$4x = y-1$

$8x = 2y-2$

$8x = y+1$

so $2y-2 = y+1$

$2y - y = 1+2$

$y = 3$

$4x = 3-1$

$4x = 2$

$x = \frac{1}{2}$

Remember to check

$4^1 = 2^2$ ✓

$9^2 = 3^4$ ✓

a Given that $y = 3^x$ express 3^{2x+2} in terms of y.

b Hence, or otherwise, solve the equation

$$3^{2x+2} - 10(3^x) + 1 = 0.$$

$3^{2x+2} = 3^{2x} \times 3^2 = 9(3^{2x}) = 9(3^x)^2 = 9y^2$

b)

$3^{2x+2} - 10(3^x) + 1 = 0$

This ends up looking like a quadratic

$9y^2 - 10y + 1 = 0$

$(9y - 1)(y - 1)$

$9y - 1 = 0$ $\quad$ $y - 1 = 0$

$y = 1/9$ $\quad$ $y = 1$

$3^x = 1/9$ $\quad$ $3^x = 1$

$x = -2$ $\quad$ $x = 0$

Check these!