Find

$$\int \frac{3x^4 - 4}{2x^3}\, dx$$

writing your answer in simplest form.

(4)

Find the value of the constant k, $0 < k < 9$, such that

$$\int_k^9 \frac{6}{\sqrt{x}}\,\mathrm{d}x = 20$$

(4)

Given that k is a positive constant and $\int_1^k \left(\frac{5}{2\sqrt{x}} + 3 \right) \mathrm{d}x = 4$

(a) show that $3k + 5\sqrt{k} - 12 = 0$

(4)

(b) Hence, using algebra, find any values of k such that

$$\int_1^k \left(\frac{5}{2\sqrt{x}} + 3 \right) \mathrm{d}x = 4$$

(4)

(a) Given that k is a constant, find

$$\int \left(\frac{4}{x^3} + kx \right) \mathrm{d}x$$

simplifying your answer.

(3)

(b) Hence find the value of k such that

$$\int_{0.5}^{2} \left(\frac{4}{x^3} + kx \right) \mathrm{d}x = 8$$

(3)

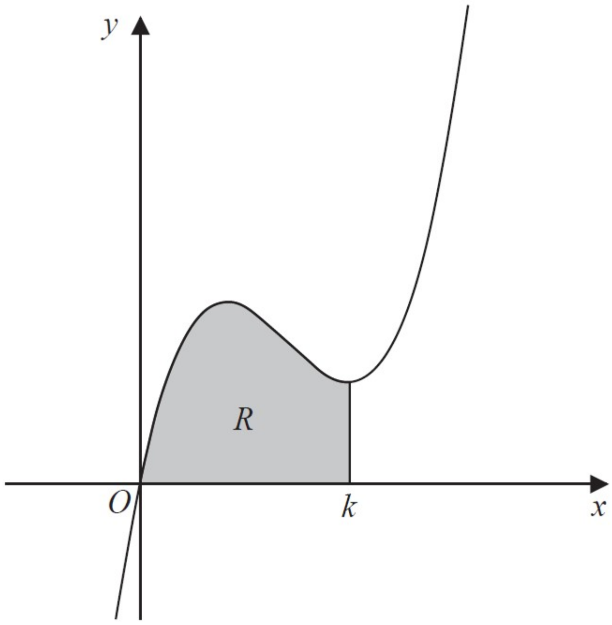

Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2x^3 - 17x^2 + 40x$$

The curve has a minimum turning point at $x = k$.

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = k$.

Show that the area of R is $\dfrac{256}{3}$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(7)

Find

$$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)\mathrm{d}x$$

giving your answer in its simplest form.

(4)

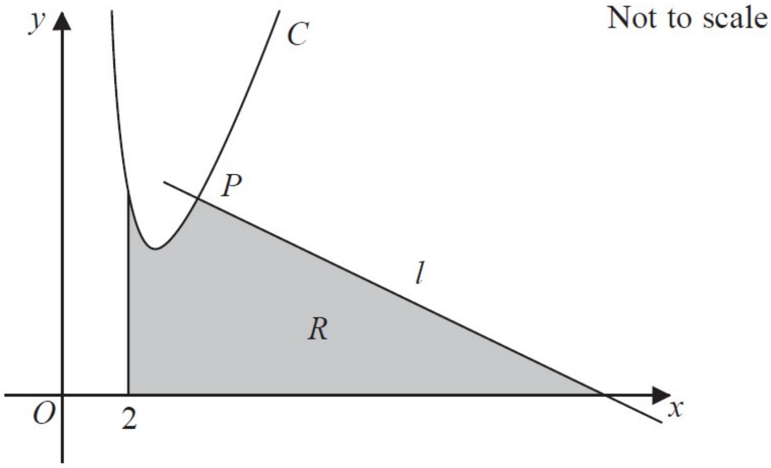

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{32}{x^2} + 3x - 8, \qquad x > 0$$

The point P (4, 6) lies on C.
The line l is the normal to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the line l, the curve C, the line with equation $x = 2$ and the x-axis.

Show that the area of R is 46
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

Given that

$$f(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0$$

show that $\int_1^{2\sqrt{2}} f(x)\,dx = 16 + 3\sqrt{2}$

(5)

Find

$$\int \left(2x^5 - \frac{1}{4x^3} - 5 \right) \mathrm{d}x$$

giving each term in its simplest form.

(4)

Find

$$\int\left(2x^4 - \frac{4}{\sqrt{x}} + 3\right)\mathrm{d}x$$

giving each term in its simplest form.

(4)

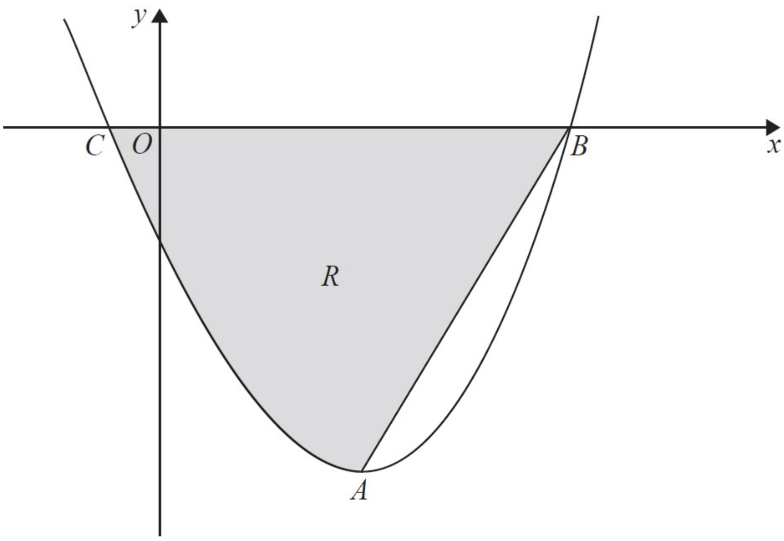

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 4x^3 + 9x^2 - 30x - 8, \quad -0.5 \leqslant x \leqslant 2.2$$

The curve has a turning point at the point A.

(a) Using calculus, show that the x coordinate of A is 1

(3)

The curve crosses the x-axis at the points $B(2, 0)$ and $C\left(-\frac{1}{4}, 0\right)$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the line AB, and the x-axis.

(b) Use integration to find the area of the finite region R, giving your answer to 2 decimal places.

(7)

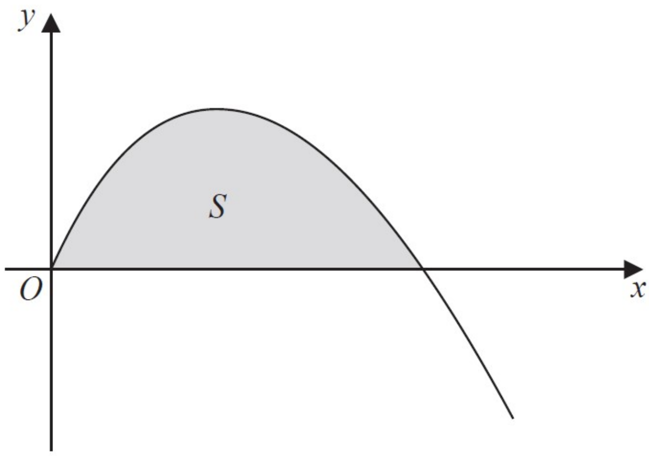

Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 3x - x^{\frac{3}{2}}, \qquad x \geqslant 0$$

The finite region S, bounded by the x-axis and the curve, is shown shaded in Figure 3.

(a) Find

$$\int \left(3x - x^{\frac{3}{2}}\right) \mathrm{d}x$$

(3)

(b) Hence find the area of S.

(3)