Find

$$\int \frac{3x^4 - 4}{2x^3}\, dx$$

writing your answer in simplest form.

(4)

$$\int \frac{3x^4}{2x^3} - \frac{4}{2x^3} \quad dx$$

$$\int \frac{3}{2}x - 2x^{-3} \; dx$$

$$\frac{3}{4}x^2 - \frac{2x^{-2}}{-2} + c$$

$$\underline{\underline{\frac{3}{4}x^2 + x^{-2} + c}}$$

Find the value of the constant k, $0 < k < 9$, such that

$$\int_k^9 \frac{6}{\sqrt{x}}\,dx = 20$$

(4)

$$\int_k^9 6x^{-\frac{1}{2}}\,dx = 20$$

$$\left[\frac{6x^{\frac{1}{2}}}{1/2}\right]_k^9 = 20$$

$$\left[12x^{\frac{1}{2}}\right]_k^9 = 20$$

$$\left(12(9)^{\frac{1}{2}}\right) - \left(12(k)^{\frac{1}{2}}\right) = 20$$

$$36 - 12\sqrt{k} = 20$$

$$16 = 12\sqrt{k}$$

$$\frac{4}{3} = \sqrt{k}$$

$$\frac{16}{9} = k$$

Given that k is a positive constant and $\int_1^k \left(\frac{5}{2\sqrt{x}} + 3\right) dx = 4$

(a) show that $3k + 5\sqrt{k} - 12 = 0$

(4)

(b) Hence, using algebra, find any values of k such that

$$\int_1^k \left(\frac{5}{2\sqrt{x}} + 3\right) dx = 4$$

(4)

a/ $\int_1^k \left(\frac{5}{2} x^{-\frac{1}{2}} + 3\right) dx = 4$

$\left[5x^{\frac{1}{2}} + 3x + c \right]_1^k = 4$

$\left(5k^{\frac{1}{2}} + 3k + c\right) - \left(5(1)^{\frac{1}{2}} + 3(1) + c\right) = 4$

$\left(5k^{\frac{1}{2}} + 3k + c\right) - \left(5 + 3 + c\right) = 4$

$5k^{\frac{1}{2}} + 3k - 8 = 4$

$5\sqrt{k} + 3k - 12 = 0$

$3k + 5\sqrt{k} - 12 = 0$

let $\sqrt{k} = x$ $\quad 3x^2 + 5x - 12 = 0$

$(3x - 4)(x + 3)$

$x = \frac{4}{3} \quad x = -3$

$\sqrt{k} = \frac{4}{3} \quad \sqrt{k} = -3$

$\underline{\underline{k = \frac{16}{9}}} \quad \times$

(a) Given that k is a constant, find

$$\int\left(\frac{4}{x^3} + kx\right)\mathrm{d}x$$

simplifying your answer.

(3)

(b) Hence find the value of k such that

$$\int_{0.5}^{2}\left(\frac{4}{x^3} + kx\right)\mathrm{d}x = 8$$

(3)

a) $\int 4x^{-3} + kx \; dx$

$-2x^{-2} + \frac{1}{2}kx^2 + c$

b) $\left[-2x^{-2} + \frac{1}{2}kx^2\right]_{0.5}^{2} = 8$

$\left(-2(2)^{-2} + \frac{1}{2}k(2)^2\right) - \left(-2(0.5)^{-2} + \frac{1}{2}k(0.5)^2\right) = 8$

$-\frac{1}{2} + 2k - \left(-8 + \frac{1}{8}k\right) = 8$

$-\frac{1}{2} + 2k + 8 - \frac{1}{8}k = 8$

$\frac{15}{8}k = \frac{1}{2}$

$k = \frac{8}{30}$

$= \frac{4}{15}$

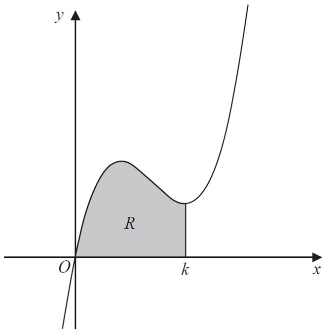

Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2x^3 - 17x^2 + 40x$$

The curve has a minimum turning point at $x = k$.

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = k$.

Show that the area of R is $\frac{256}{3}$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(7)

Min point where $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 6x^2 - 34x + 40$$

$$6x^2 - 34x + 40 = 0$$

$$3x^2 - 17x + 20 = 0$$

$$\cancel{(3x - 20)(x - 1) = 0}$$

$$(3x - 5)(x - 4) = 0$$

$$x = \frac{5}{3} \quad x = 4$$

$$k = 4$$

$$\int_0^4 2x^3 - 17x^2 + 40x \, dx =$$

$$\left[\frac{1}{2}x^4 - \frac{17}{3}x^3 + 20x^2\right]_0^4$$

$$\frac{1}{2}(4)^4 - \frac{17}{3}(4)^3 + 20(4)^2 - 0$$

$$= \frac{256}{3}$$

Find

$$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)\mathrm{d}x$$

giving your answer in its simplest form.

(4)

$$\frac{2}{3}x^3 - 6x^{\frac{1}{2}} + 1$$

$$\frac{2}{12}x^4 - \frac{6x^{3/2}}{3/2} + x + c$$

$$\frac{1}{6}x^4 - 4x^{\frac{3}{2}} + x + c$$

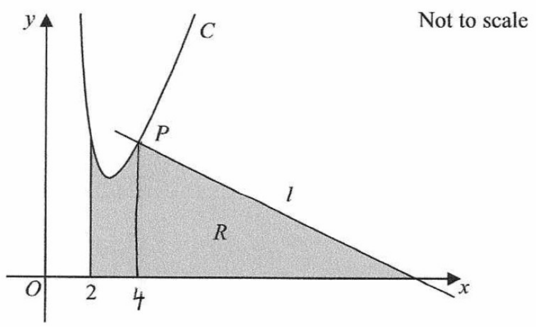

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{32}{x^2} + 3x - 8, \qquad x > 0$$

The point P (4, 6) lies on C.
The line l is the normal to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the line l, the curve C, the line with equation $x = 2$ and the x-axis.

Show that the area of R is 46
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

$y = 32x^{-2} + 3x - 8$

$\frac{dy}{dx} = -64x^{-3} + 3$

when $x = 4$ $\quad \frac{dy}{dx} = -64(4)^{-3} + 3$

$= 2$

perp $m = -\frac{1}{2}$

$y = -\frac{1}{2}x + c \qquad (4, 6)$

$6 = -\frac{1}{2}(4) + c$

$6 = -2 + c$

$c = 8$

Equation of l : $y = -\frac{1}{2}x + 8$

l crosses x when $y = 0$ $\quad 0 = -\frac{1}{2}x + 8$

$\underline{\underline{x = 16}}$

R = area of triangle + area under curve.

area of triangle $= \frac{1}{2}(12)(6)$

$= \underline{\underline{36}}$

area under curve $= \int_2^4 32x^{-2} + 3x - 8 \; dx$

$= \left[-32x^{-1} + \frac{3}{2}x^2 - 8x\right]_2^4$

$\left(-32(4)^{-1} + \frac{3}{2}(4)^2 - 8(4)\right) - \left(-32(2)^{-1} + \frac{3}{2}(2)^2 - 8(2)\right)$

$-16 - (-26)$

$= \underline{\underline{10}}$

$R = 36 + 10$

$= \underline{\underline{46}}$

Given that

$$f(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0$$

show that $\int_1^{2\sqrt{2}} f(x)\,dx = 16 + 3\sqrt{2}$

(5)

$$\int_1^{2\sqrt{2}} 2x + 3 + 12x^{-2} \; dx$$

$$\left[x^2 + 3x - 12x^{-1} \right]_1^{2\sqrt{2}}$$

$$\left[(2\sqrt{2})^2 + 3(2\sqrt{2}) - 12(2\sqrt{2})^{-1} \right] - \left[(1)^2 + 3(1) - 12(1)^{-1} \right]$$

$$\left(8 + 3\sqrt{2} \right) - \left(-8 \right)$$

$$\underline{\underline{16 + 3\sqrt{2}}}$$

Find

$$\int\left(2x^5 - \frac{1}{4x^3} - 5\right)\mathrm{d}x$$

giving each term in its simplest form.

(4)

$$\int 2x^5 - \frac{1}{4}x^{-3} - 5 \; dx$$

$$\frac{2x^6}{6} - \frac{\frac{1}{4}x^{-2}}{-2} - 5x + c$$

$$\frac{1}{3}x^6 + \frac{1}{8}x^{-2} - 5x + c$$

Find

$$\int \left(2x^4 - \frac{4}{\sqrt{x}} + 3 \right) \mathrm{d}x$$

giving each term in its simplest form.

(4)

$$\int 2x^4 - 4x^{-\frac{1}{2}} + 3 \quad dx$$

$$\frac{2x^5}{5} - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} + 3x + c$$

$$\frac{2}{5}x^5 - 8x^{\frac{1}{2}} + 3x + c$$

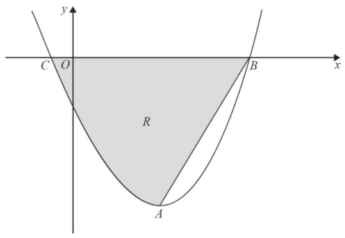

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 4x^3 + 9x^2 - 30x - 8, \quad -0.5 \leqslant x \leqslant 2.2$$

The curve has a turning point at the point A.

(a) Using calculus, show that the x coordinate of A is 1

(3)

The curve crosses the x-axis at the points $B(2, 0)$ and $C\left(-\frac{1}{4}, 0\right)$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the line AB, and the x-axis.

(b) Use integration to find the area of the finite region R, giving your answer to 2 decimal places.

(7)

a/ turning point where $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 12x^2 + 18x - 30$$

$$12x^2 + 18x - 30 = 0$$
$$6x^2 + 9x - 15 = 0$$
$$2x^2 + 3x - 5 = 0$$
$$(2x + 5)(x - 1)$$
$$x = -\frac{5}{2} \qquad x = 1$$

$-0.5 \leq x \leq 2.2 \quad \therefore \quad \underline{\underline{x = 1}}$

b/ $\int_{-\frac{1}{4}}^{1} 4x^3 + 9x^2 - 30x - 8 \, dx$

$$\left[x^4 + 3x^3 - 15x^2 - 8x \right]_{-1/4}^{1}$$

$$\left[(1)^4 + 3(1)^3 - 15(1)^2 - 8(1)\right] - \left[\left(-\frac{1}{4}\right)^4 + 3\left(-\frac{1}{4}\right)^3 - 15\left(-\frac{1}{4}\right)^2 - 8\left(-\frac{1}{4}\right)\right]$$

$$(-19) - \left(\frac{261}{256}\right)$$

$= -20.02$

$= \underline{\underline{20.02 \text{ units}^2}}$

when $x = 1$ $\quad y = 4(1)^3 + 9(1)^2 - 30(1) - 8$
$= -25$

Area of triangle $= \frac{1}{2}(1)(25)$
$= 12.5 \text{ units}^2$

Total Area $= 20.02 + 12.5$
$= \underline{\underline{32.52}} \text{ units}^2$

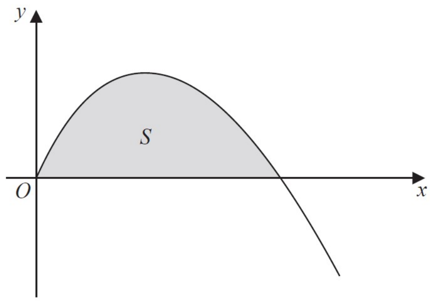

Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 3x - x^{\frac{3}{2}}, \qquad x \geqslant 0$$

The finite region S, bounded by the x-axis and the curve, is shown shaded in Figure 3.

(a) Find

$$\int \left(3x - x^{\frac{3}{2}}\right) dx$$

(3)

(b) Hence find the area of S.

a) $\int 3x - x^{\frac{3}{2}} \, dx$

$\frac{3x^2}{2} - \frac{x^{\frac{5}{2}}}{5/2} + c$

$\frac{3}{2}x^2 - \frac{2}{5}x^{\frac{5}{2}} + c$

b/ crosses x when $y=0$

$0 = 3x - x^{3/2}$

$0 = x(3 - x^{\frac{1}{2}})$

$x = 0 \quad x^{\frac{1}{2}} = 3$

$x = 9$

$\left[\frac{3}{2}x^2 - \frac{2}{5}x^{\frac{5}{2}} + c\right]_0^9$

$\left(\frac{3}{2}(9)^2 - \frac{2}{5}(9)^{\frac{5}{2}}\right) - (0)$

$= 24.3$ units^2