Solve the equation

$$2\log_2(x) - \log_2(5) = 1$$

$$2\log_2(x) - \log_2(5) = 1$$

$$\log_2(x^2) - \log_2(5) = 1$$

$$\log_2\left(\frac{x^2}{5}\right) = 1$$

$$\frac{x^2}{5} = 2^1$$

$$x^2 = 10$$

$$x = \pm\sqrt{10}$$

$\log_n a = b$
means
$a = n^b$

check using $\boxed{\log_{\square}\blacksquare}$ button

Solve the equation

$$\log_3(x) + \log_3(4) = 2$$

$$\log_3(x) + \log_3(4) = 2$$

$$\log_3(4x) = 2$$

$$4x = 3^2$$

$$x = 9/4$$

check using

$\boxed{\log_{\square}\blacksquare}$ button

a Solve the equation

$$\log_3 (x + 1) - \log_3 (x - 2) = 1. \qquad \textbf{(3)}$$

b Find, in terms of logarithms to the base 10, the exact value of x such that

$$3^{2x+1} = 2^{x-4}. \qquad \textbf{(3)}$$

a) $\log_3(x+1) - \log_3(x-2) = 1$

$$\log_3\left(\frac{x+1}{x-2}\right) = 1$$

$$\frac{x+1}{x-2} = 3^1$$

$$\frac{x+1}{x-2} = \frac{3}{1}$$

$$x+1 = 3x-6$$

$$7 = 2x \qquad x = 7/2$$

Check $\boxed{\log_{\square}\blacksquare}$

b) $3^{2x+1} = 2^{x-4}$

$$\log_{10} 3^{2x+1} = \log_{10} 2^{x-4}$$

$$(2x+1)\log_{10}3 = (x-4)\log_{10}2$$

$$2x\log_{10}3 - x\log_{10}2 = -\log_{10}3 - 4\log_{10}2$$

$$\log_{10}3 + 4\log_{10}2 = x\log_{10}2 + 2x\log_{10}3$$

$$\log_{10}3 + \log_{10}2^4 = x(\log_{10}2 + \log_{10}9)$$

$$\frac{\log_{10}48}{\log_{10}18} = x$$

Express as a single logarithm to base a

$$2\log_a(x+1) - \log_a(4)$$

$2\log_a(x+1) - \log_a(4)$

$\log_a(x+1)^2 - \log_a(4)$

$\log_a\left(\frac{(x+1)^2}{4}\right)$

Giving your answers to 2 decimal places, solve the simultaneous equations

$$e^{2y} = x + 1$$
$$\ln(x - 2) = 2y - 1$$

Solve by substitution of $2y = \ln(x-2) + 1$

into first equation

$$e^{\ln(x-2)+1} = x + 1$$

$$e^{\ln(x-2)} \times e^{1} = x + 1$$

$$(x-2) \times e = x + 1$$

$$xe - 2e = x + 1$$

$$x(e-1) = 1 + 2e$$

$$x = \frac{1+2e}{e-1} = 3.746$$

$$2y = \ln(x-2) + 1$$

$$2y = \ln(1.746) + 1$$

$$y = \frac{\ln(1.746) + 1}{2}$$

$$y = 0.7787$$

Ans $x = 3.75$ & $y = 0.78$ ✓

Solve the equation

$$\ln(2x + 5) = 1$$

$\ln(2x+5) = 1$

Take e of both sides

$e^{\ln(2x+5)} = e^1$

$2x+5 = e$

$2x = e - 5$

$x = \dfrac{e-5}{2}$ ✓

check on calculator

Solve the equation, giving your answers in exact form.

$$2e^{y} + 15e^{-y} = 11$$

$$2e^{y} + \frac{15}{e^{y}} = 11$$

Nasty looking but if I multiply all by e^{y}

$$2e^{2y} + 15 = 11e^{y}$$

$$2e^{2y} - 11e^{y} + 15 = 0$$

$$2(e^{y})^{2} - 11(e^{y}) + 15 = 0$$

Quadratic in e^{y}

similar

$$2x^{2} - 11x + 15 = 0$$

$$(2x - 5)(x - 3) = 0$$

$2x - 5 = 0$ $\qquad$ $x - 3 = 0$

$x = 5/2$ $\qquad$ $x = 3$

$e^{y} = 5/2$ $\qquad$ $e^{y} = 3$

$y = \ln(5/2)$ $\qquad$ $y = \ln(3)$

2 answers

Ans $\ln(\frac{5}{2})$ & $\ln(3)$

Solve each equation, giving your answers correct to 2 decimal places.

a $e^{2x} - 5.7e^{-x} = 0$

b $\ln x - \ln(x-1) = \frac{1}{2}$

$$e^{2x} - 5.7e^{-x}$$

$$\frac{e^{2x}}{1} = \frac{5.7}{e^{x}}$$

$$e^{3x} = 5.7$$

found by cross-multiplying.

$$\ln(e^{3x}) = \ln 5.7$$

$$3x = \ln 5.7$$

$$x = \frac{\ln 5.7}{3}$$

$$x = 0.580$$

$$x = 0.58$$

$$\ln(x) - \ln(x-1) = \frac{1}{2}$$

$$\ln\left(\frac{x}{x-1}\right) = \frac{1}{2}$$

Take e on both sides

$$e^{\ln\left(\frac{x}{x-1}\right)} = e^{1/2}$$

$$\frac{x}{x-1} = e^{1/2}$$

cross multiply

$$\frac{x}{x-1} = \frac{e^{1/2}}{1}$$

$$x = (x-1)(e^{1/2})$$

$$x = xe^{1/2} - e^{1/2}$$

$$x(1 - e^{1/2}) = -e^{1/2}$$

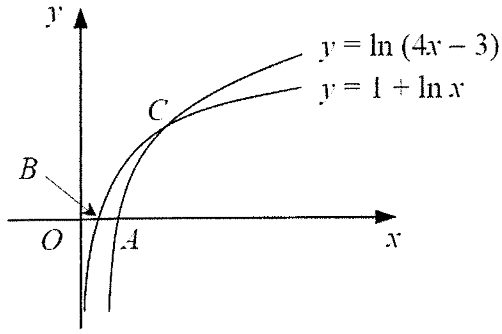
$$x = \frac{-e^{1/2}}{1-e^{1/2}}$$

$$x = 2.5414$$

$$x = 2.54$$

Answers

0.58 & 2.54



The diagram shows the curves $y = \ln(4x-3)$ and $y = 1 + \ln x$ which cross the x-axis at the points A and B respectively.

a Find the coordinates of A and B. **(4)**

The two curves intersect at the point C.

b Find the exact x-coordinate of C, giving your answer in terms of e. **(4)**

Point A

Curve $y = \ln(4x-3)$

But $y=0$ $0 = \ln(4x-3)$

$e^0 = 4x-3$

$1 = 4x-3$

$4 = 4x$

$1 = x$

Point $A = (1,0)$

Point B

Curve $y = 1 + \ln x$

But $y=0$

$0 = 1 + \ln x$

$-1 = \ln x$

$e^{-1} = x$

0.368 $\frac{1}{e}$

Point $B = (\frac{1}{e}, 0)$

Intersect at C

$\ln(4x-3) = 1 + \ln(x)$

$\ln(4x-3) - \ln(x) = 1$

$\ln\left(\frac{4x-3}{x}\right) = 1$

$\frac{4x-3}{x} = e^1$

$4x-3 = xe$

$x(4-e) = 3$

$x = \frac{3}{4-e}$

$y = 1 + \ln\left(\frac{3}{4-e}\right)$

$y =$

$\left(\frac{3}{4-e}, 1 + \ln\left(\frac{3}{4-e}\right)\right)$

Giving your answers in exact form, solve the equations

a $\ln(4x - 1) = 2$. **(3)**

b $7 - e^{1-3y} = 0$. **(3)**

$\ln(4x-1) = 2$

$e^{\ln(4x-1)} = e^2$

$4x - 1 = e^2$

$4x = e^2 + 1$

$x = \dfrac{e^2+1}{4}$

You can check!

Use $\boxed{\ln}$ button

$7 - e^{1-3y} = 0$

$e^{1-3y} = 7$

$\ln(e^{1-3y}) = \ln 7$

$1 - 3y = \ln 7$

$1 - \ln 7 = 3y$

$\dfrac{1-\ln 7}{3} = y$

Again check it!

a Simplify

$$\frac{x^2-4x+3}{x^2+x-2}.$$ **(3)**

b Solve the equation

$$\ln(x^2-4x+3) = 1 + \ln(x^2+x-2),$$

giving your answer in terms of e. **(4)**

$$\frac{x^2-4x+3}{x^2+x-2} = \frac{(x-3)\cancel{(x-1)}}{(x+2)\cancel{(x-1)}} = \frac{x-3}{x+2}$$

$$\ln(x^2-4x+3) = 1 + \ln(x^2+x-2)$$

$$\ln(x^2-4x+3) - \ln(x^2+x-2) = 1$$

$$\ln\left(\frac{x^2-4x+3}{x^2+x-2}\right) = 1$$

$$\ln\left(\frac{x-3}{x+2}\right) = 1$$

$$\frac{x-3}{x+2} = e^1$$

$$x-3 = e(x+2)$$

$$x - ex = 2e+3$$

$$x(1-e) = 2e+3$$

$$x = \frac{2e+3}{1-e}$$

Giving your answers to an appropriate degree of accuracy, solve the simultaneous equations

$$e^y + 5 - 9x = 0$$

$$y - \ln(x + 4) = 2$$ $\quad y = 2 + \ln(x+4)$ **(7)**

$$e^{2+\ln(x+4)} + 5 - 9x = 0$$

$$e^2 \times e^{\ln(x+4)} + 5 - 9x = 0$$

$$e^2(x+4) + 5 - 9x = 0$$

$$x(e^2 - 9) = -5 - 4e^2$$

$$5 + 4e^2 = x(9 - e^2)$$

$$\frac{5+4e^2}{9-e^2} = x$$

$$21.45 = x$$

Then $y = 2 + \ln(x+4)$

$$y = 2 + \ln(21.45 + 4)$$

$$y = 5.2367$$

$$5.24$$

Ans $x = 21.45$ & $y = 5.24$

(2dec. pl.)

a Given that $t = \ln x$, find expressions in terms of t for

i $\ln \sqrt{x}$,

ii $\ln (e^2 x)$. **(4)**

b Hence, or otherwise, solve the equation

$$5 + \ln \sqrt{x} = \ln (e^2 x).$$ **(3)**

(i)

$t = \ln x$

$\ln \sqrt{x} = \ln x^{\frac{1}{2}} = \frac{1}{2} \ln x = \frac{1}{2} t$

Ans $\frac{1}{2} t$ or $\frac{t}{2}$

(ii) $\ln (e^2 x)$

$= \ln e^2 + \ln x$

$= 2 \ln e + \ln x$

$= 2 \times 1 + t$

$= 2 + t$

Ans $2 + t$

(b) $5 + \ln \sqrt{x} = \ln (e^2 x)$

$5 + \frac{t}{2} = 2 + t$

$5 - 2 = t - \frac{t}{2}$

$3 = \frac{t}{2}$

$6 = t$

Ans $t = 6$

$6 = \ln x$

$e^6 = x$

Ans $x = e^6$

Superb. It checks out!