Solve the simultaneous equations
Give your answers to 3 significant figures — clue

$$x^2 + y^2 = 20$$

$$2x + y = 3$$

$x^2 + y^2 = 20$

$2x + y = 3 \longrightarrow y = 3 - 2x$ Substitute this into the other eq.

$x^2 + (3-2x)^2 = 20$

$x^2 + 9 - 12x + 4x^2 = 20$

$5x^2 - 12x - 11 = 0$

3 sig. fig. is a clue that we use the quadratic formula $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

gives $x = \dfrac{6 \pm \sqrt{91}}{5}$

$x = 3.11$ & -0.71

then put these in to find y values

$y = 3 - 2(3.11)$ $y = 3 - 2(-0.71)$

$y = -3.22$ $y = 4.42$

Ans $x = 3.11$ & $y = -3.22$

AND $x = -0.71$ & $y = 4.42$

$$x^2 - 2xy - y^2 = 7$$

$$x + y = 1$$

$$y = 1 - x$$

Substitute $y = 1 - x$ into the other equation

$$x^2 - 2x(1-x) - (1-x)^2 = 7$$

$$x^2 - 2x + 2x^2 - 1 + 2x - x^2 = 7$$

$$2x^2 - 8 = 0$$

$$2x^2 = 8$$

$$x^2 = 4$$

$$x = \pm 2$$

when $x = +2$

$$y = 1 - 2$$

$$y = -1$$

when $x = -2$

$$y = 1 - (-2)$$

$$y = 1 + 2$$

$$y = 3$$

Answer

$x = 2$ and $y = -1$

AND $x = -2$ and $y = 3$

The straight line l is a tangent to the curve $y = x^2 - 5x + 3$ at the point A on the curve.

Given that l is parallel to the line $3x + y = 0$,

a find the coordinates of the point A,

b find the equation of the line l in the form $y = mx + c$.

$y=$ curve $\quad y = x^2 - 5x + 3$

$\frac{dy}{dx}$ = gradient $= 2x - 5$ at the x values

line $3x + y = 0$ means $y = -3x$

line l has gradient -3

so $\frac{dy}{dx}$ = gradient $= -3 = 2x - 5$

$-3 + 5 = 2x$

$2 = 2x$

$1 = x$

Point A $(1, -1)$

Equation $y = mx + c$

$y = 3x + c$

Substitute point $(1, -1)$ in to get c

$-1 = 3(1) + c$

$-4 = c$

Ans $y = 3x - 4$

$y = x^3 - 4x^2 - 3x + 9$

(a) Find $\frac{dy}{dx}$

(b) Find the range values of x for which y is increasing

$y = x^3 - 4x^2 - 3x + 9$ $\quad y =$ curve

$\frac{dy}{dx} = 3x^2 - 8x - 3$ $\quad \frac{dy}{dx} =$ gradient

When is y increasing?

When $\frac{dy}{dx} > 0$ $\quad$ Gradient > 0

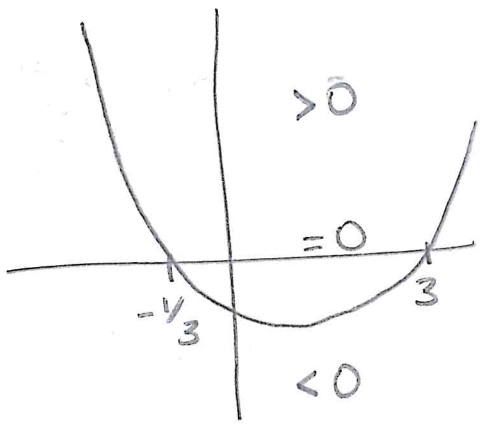
$3x^2 - 8x - 3 > 0$

You can solve this on new white calculator

B: INEQUALITY
DEGREE 2
Quadratic Inequality

or factorise

$(3x + 1)(x - 3) > 0$



$3x + 1 = 0$ $\quad x - 3 = 0$

$x = -1/3$ $\quad x = 3$

$3x^2 - 8x - 3 > 0$

is above x axis

Ans $\quad x < -1/3$ and $x > 3$

Solve each pair of simultaneous equations.

$$x - \frac{1}{y} - 4y = 0 \longrightarrow$$ get rid of $\frac{1}{y}$ by multiplying by y

$$xy - 1 - 4y^2 = 0$$

$$x - 6y - 1 = 0$$

$$x = 6y + 1$$

Now solve simult. eqs

$$\boxed{\begin{aligned} xy - 1 - 4y^2 &= 0 \\ x &= 6y+1 \end{aligned}}$$

Substitute $x = 6y+1$ into the other equation

$$xy - 1 - 4y^2 = 0$$

$$(6y+1)y - 1 - 4y^2 = 0$$

$$6y^2 + y - 1 - 4y^2 = 0$$

$$2y^2 + y - 1 = 0$$

$$(2y-1)(y+1) = 0$$

$$2y - 1 = 0 \qquad\qquad y + 1 = 0$$

$$y = \tfrac{1}{2} \qquad\qquad y = -1$$

use $x = 6y+1$ to get x values

$$x = 6(\tfrac{1}{2}) + 1. \qquad\qquad x = 6(-1) + 1$$

$$x = 3 + 1 \qquad\qquad x = -6 + 1$$

$$x = 4 \qquad\qquad x = -5$$

Answers $x = 4$ & $y = \tfrac{1}{2}$ AND $x = -5$ & $y = -1$

Solve the simultaneous equations

$$3^{x-1} = 9^{2y}$$
$$8^{x-2} = 4^{1+y}$$

$3, 9$ are connected

$8 \,\&\, 4$ are connected

$$3^{x-1} = 9^{2y}$$
$$3^{x-1} = (3^2)^{2y}$$
$$3^{x-1} = 3^{4y}$$
$$\boxed{x-1 = 4y}$$

$$8^{x-2} = 4^{1+y}$$
$$(2^3)^{x-2} = (2^2)^{1+y}$$
$$2^{3x-6} = 2^{2+2y}$$
$$\boxed{3x-6 = 2+2y}$$

$$x - 4y = 1$$
$$3x - 2y = 8$$

You can solve these on calculator

A: EQUATION/FUNCTION
1: SIMUL EQUATIONS

or

$$3x - 2y = 8$$
$$x - 4y = 1$$

$$6x - \cancel{4y} = 16$$
$$x - \cancel{4y} = 1$$
$$5x = 15$$
$$x = 3$$

then $y = 1/2$

Don't forget to check!

Find $\dfrac{dy}{dx}$

$y = 3x^2 + \sqrt[3]{x} = 3x^2 + x^{1/3}$

$\dfrac{dy}{dx} = 6x + \frac{1}{3} x^{-2/3}$

$\dfrac{dy}{dx} = 6x + \dfrac{1}{3x^{2/3}}$

$\dfrac{dy}{dx} = 6x + \dfrac{1}{3\sqrt[3]{x^2}}$

$y = \dfrac{4x^3 + x}{x^2}$ can't do that !! split up the fraction

$\dfrac{4x^3}{x^2} + \dfrac{x}{x^2}$

$y = 4x + \dfrac{1}{x}$

$y = 4x + x^{-1}$

$\dfrac{dy}{dx} = 4 - 1x^{-2}$

$= 4 - \dfrac{1}{x^2}$

$y = \sqrt{x}\,(x - 4)$ can't do that!!! multiply out the ( )

$x^{1/2}(x - 4)$

$x^{3/2} - 4x^{1/2}$

$y = x^{3/2} - 4x^{1/2}$

$\dfrac{dy}{dx} = \dfrac{3}{2}x^{3/2} - 2x^{-1/2}$

$= \dfrac{3\sqrt{x^3}}{2} - \dfrac{2}{\sqrt{x}}$

The curve with equation $y = x^3 - 4x^2 + 3x$ crosses the x-axis at the points A, B and C.

a Find the coordinates of the points A, B and C.

b Find the gradient of the curve at each of the points A, B and C.

crosses x axis when $y = 0$

$$0 = x^3 - 4x^2 + 3x$$

$$0 = x(x^2 - 4x + 3)$$

$$0 = x\,(x-3)(x-1)$$

$x = 0$ $\quad$ $x = 3$ $\quad$ $x = 1$

A is $(0,0)$ $\quad$ B is $(1,0)$ $\quad$ $C = (3,0)$

Gradient at point

$$y = \text{curve} = x^3 - 4x^2 + 3x$$

$$\frac{dy}{dx} = \text{gradient} = 3x^2 - 8x + 3$$

when $x = 0$ $\quad \frac{dy}{dx} = 3(0)^2 - 8(0) + 3 = 3$

when $x = 1$ $\quad \frac{dy}{dx} = 3(1)^2 - 8(1) + 3 = -2$

when $x = 3$ $\quad \frac{dy}{dx} = 3(3)^2 - 8(3) + 3 = 6$

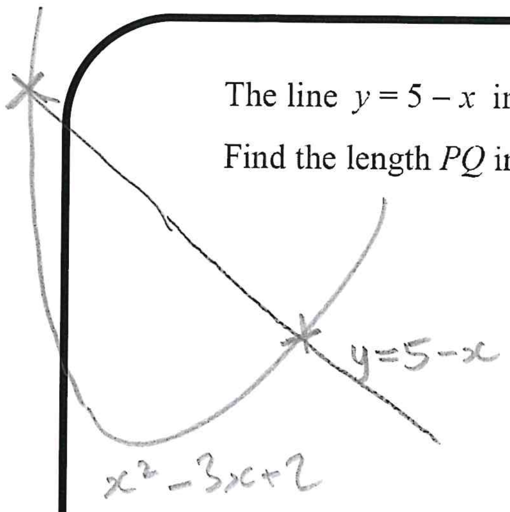
You can do this on your calulator

$\boxed{\frac{d}{dx}\square}$ button

$$\left.\frac{d}{dx}\left(x^3 - 4x^2 + 3x\right)\right|_{x=0}$$

The line $y = 5 - x$ intersects the curve $y = x^2 - 3x + 2$ at the points P and Q.

Find the length PQ in the form $k\sqrt{2}$.



simultaneous equations

$$y = 5 - x$$
$$y = x^2 - 3x + 2$$

$$5 - x = x^2 - 3x + 2$$
$$0 = x^2 - 2x - 3$$
$$0 = (x - 3)(x + 1)$$

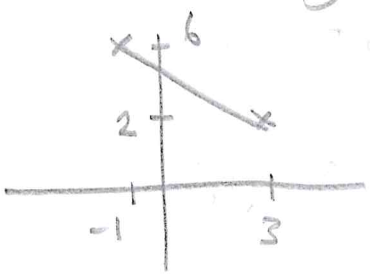
$x - 3 = 0$ $\quad$ $x + 1 = 0$

$x = 3$ $\quad$ $x = -1$

When $x = 3$, $y = 2$ $\quad$ when $x = -1$, $y = 6$

$P = (3, 2)$ $\quad$ $Q = (-1, 6)$

What is length



$$\text{Length} = \sqrt{4^2 + 4^2}$$
$$= \sqrt{16 + 16}$$
$$= \sqrt{32}$$

Given that

$$(A + 2\sqrt{3})(B - \sqrt{3}) \equiv 9\sqrt{3} - 1$$

find the values of the integers A and B.

$(A + 2\sqrt{3})(B - \sqrt{3})$ FOIL

$AB - A\sqrt{3} + 2B\sqrt{3} - 6$

$(AB - 6) + (2B - A)\sqrt{3}$

$2B - A = 9$

$AB - 6 = -1$

$AB = 5$

$A = \frac{5}{B}$

$2B - \frac{5}{B} = 9$

$2B^2 - 5 = 9B$

$2B^2 - 9B - 5 = 0$

$(2B + 1)(B - 5) = 0$

$2B + 1 = 0$ $\quad$ $B = 5$

$B = -\frac{1}{2}$

not an integer ✗

Then $A = \frac{5}{B}$

$A = \frac{5}{5}$

$A = 1$

Answer $A = 1$ and $B = 5$

A curve has the equation $y = x^2 - 3x + 4$.

a Find an equation of the normal to the curve at the point A (2, 2).

The normal to the curve at A intersects the curve again at the point B.

b Find the coordinates of the point B.

$y = x^2 - 3x + 4$ $\quad$ $y =$ curve

$\frac{dy}{dx} = 2x - 3$ $\quad$ $\frac{dy}{dx} =$ gradient

Point $x = 2$ $\quad$ $y = 2$

$\frac{dy}{dx} = 2(2) - 3$

$= 4 - 3$

$= 1$

You can do this on your calculator

$\frac{d}{dx}(x^2 - 3x + 4)\Big|_{x=2}$

Tangent $= 1$

Normal $= -1$

Equation $y = mx + c$

$y = -1x + c$

Put in $(2, 2)$

$2 = -1(2) + c$

$4 = c$ $\quad$ $y = -1x + 4$

Then simultaneous equations

$y = -x + 4$

$y = x^2 - 3x + 4$

$-x + 4 = x^2 - 3x + 4$

$0 = x^2 - 2x$

$0 = x(x - 2)$

$x = 0$ $\quad$ $x = 2$

Then $y = -4$ $\quad$ $y = -2$

Ans

$(0, -4)$ & $(2, -2)$

$y = \dfrac{(4x-1)(x+2)}{2x}$ $\quad y = \dfrac{4x^2 - x + 8x - 2}{2x} = \dfrac{4x^2 + 7x - 2}{2x}$

Find the equation of the normal at the point when $x = -2$
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.

$y = \dfrac{4x^2}{2x} + \dfrac{7x}{2x} - \dfrac{2}{2x}$

$y = 2x + \dfrac{7}{2} - \dfrac{1}{x}$ $\quad = \text{curve} = 2x + \dfrac{7}{2} - x^{-1}$

$\dfrac{dy}{dx} = 2 + 0 + x^{-2}$

$= 2 + \dfrac{1}{x^2}$

Point when $x = -2$

$m = \dfrac{dy}{dx} = 2 + \dfrac{1}{(-2)^2} = 2 + \dfrac{1}{4} = \dfrac{9}{4}$

What is y?

$y = 2(-2) + \dfrac{7}{2} - \dfrac{1}{-2} = -4 + \dfrac{7}{2} + \dfrac{1}{2}$

$= 0$

$(-2, 0)$

$y = mx + c$

$0 = \dfrac{9}{4}(-2) + c$

$0 = \dfrac{-18}{4} + c$

$\dfrac{18}{4} = c$

$y = \dfrac{9}{4}x + \dfrac{18}{4}$

$4y = 9x + 18$

$0 = 9x - 4y + 18$