Solve, for $0 \leq x < 360°$, the equation,

$$\tan^2(x) = 3$$

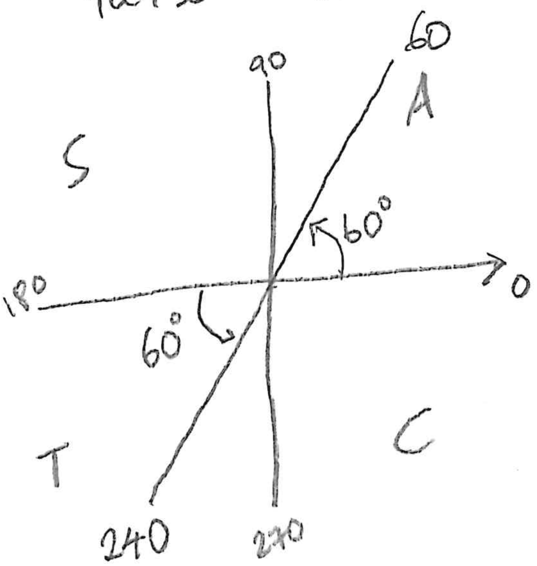
$\tan^2(x) = 3$

$\tan(x) = \sqrt[2]{3}$

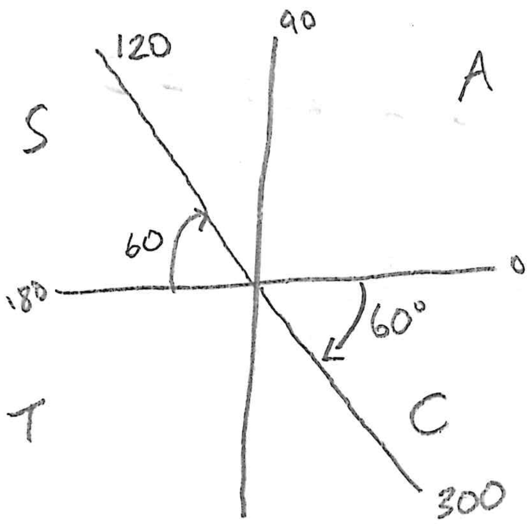
$\tan(x) = \pm\sqrt{3}$

Watch out $\pm\sqrt{}$

$\tan x = +\sqrt{3}$



$\tan x = -\sqrt{3}$



Ans $60°$, & $240°$ <u>and</u> $120°$ & $300°$

(a) Show that the equation

$$6\cos^2 x = 4 - \sin x$$

Can be written in the form

$$6\sin^2 x - \sin x - 2 = 0$$

(3)

(b) Hence solve for $0 \le x < 360°$, the equation,

$$6\cos^2 x = 4 - \sin x$$

(6)

$$6\cos^2 x = 4 - \sin x$$

$$6(1 - \sin^2 x) = 4 - \sin x$$

$$6 - 6\sin^2 x = 4 - \sin x$$

$$0 = 6\sin^2 x - \sin x - 2 \quad ✓$$

Solve $\cancel{0 = 6\sin^2 x - \sin x - 2}$

solve $6\cos^2 x = 4 - \sin x$

is exactly

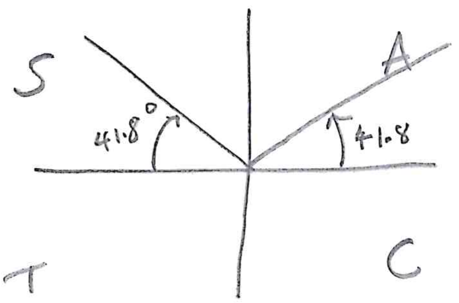
$$6\sin^2 x - \sin x - 2 = 0$$

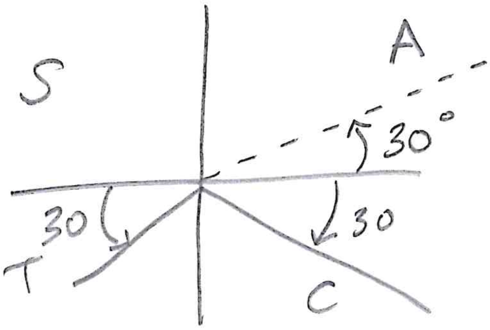
$$(3\sin x - 2)(2\sin x + 1) = 0$$

Each ( ) = 0

$$3\sin x - 2 = 0 \qquad\qquad 2\sin x + 1 = 0$$

$$\sin x = 2/3 \qquad\qquad \sin x = -1/2$$





Ans

$41.8°$ & $138.2°$ and $210°$ & $330°$

Check with calculator

Given that $x > 0$, find in the form $k\sqrt{3}$ the value of x such that

$$x(x-2) = 2(6-x).$$

$x^2 - 2x = 12 - 2x$

$x^2 = 12$

$x = \sqrt{12}$

$x = \sqrt{4}\sqrt{3}$

$x = 2\sqrt{3}$

Solve each equation, giving your answers as simply as possible in terms of surds.

$x\sqrt{18} - 4 = \sqrt{8}$

$x\sqrt{18} = 4 + \sqrt{8}$

$3\sqrt{2}x = 4 + \sqrt{8}$

$x = \frac{4 + \sqrt{8}}{3\sqrt{2}}$

$x = \frac{4\sqrt{2} + 4}{6}$ Rationalise $\times \frac{\sqrt{2}}{\sqrt{2}}$

$x = \frac{2\sqrt{2} + 2}{3}$

$x\sqrt{5} + 2 = \sqrt{20}\,(x - 1)$

$\sqrt{5}x + 2 = \sqrt{20}x - \sqrt{20}$

$2 + \sqrt{20} = \sqrt{20}x - \sqrt{5}x$

$2 + 2\sqrt{5} = 2\sqrt{5}x - \sqrt{5}x$

$2 + 2\sqrt{5} = \sqrt{5}x$

$\frac{2 + 2\sqrt{5}}{\sqrt{5}} = x$ Rationalise $\frac{\times\sqrt{5}}{\times\sqrt{5}}$

$\frac{(2 + 2\sqrt{5})\sqrt{5}}{5} = x$

$2 + \frac{2}{5}\sqrt{5} = x$ or $\frac{10 + 2\sqrt{5}}{5}$

Solve each equation for x in the interval $0 \le x \le 360°$.

$3\cos^2 x - \sin^2 x = 2$

$3\cos^2 x - (1 - \cos^2 x) = 2$

$3\cos^2 x - 1 + \cos^2 x = 2$

$4\cos^2 x = 3$

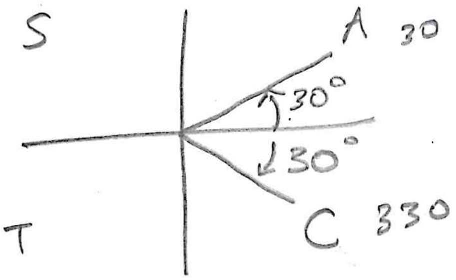
$\cos^2 x = 3/4$

$\cos x = \pm\sqrt{3/4}$

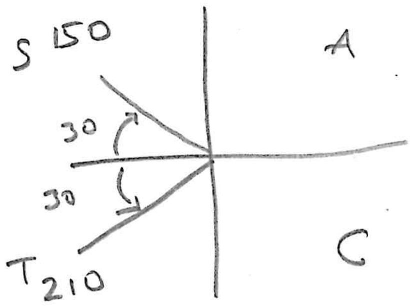
Remember $\pm$ square root answers

$\cos x = \pm \sqrt{3}/2$

$\cos x = +\sqrt{3}/2$



$\cos x = -\sqrt{3}/2$



Ans $30°, 150°, 210°, 330°$

$3 \sin x \tan x = 8$

$3\sin x \cdot \dfrac{\sin x}{\cos x} = 8$

$3\sin^2 x = 8\cos x$

$3(1 - \cos^2 x) = 8\cos x$

$0 = 3\cos^2 x + 8\cos x - 3$

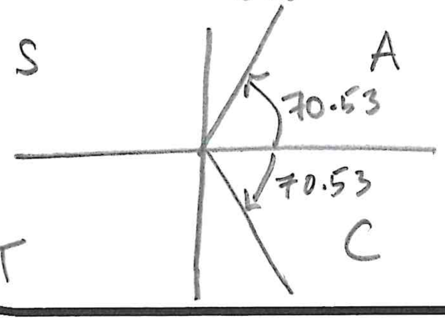
$0 = (3\cos x - 1)(\cos x + 3)$

$\cos x = 1/3$

$\cos x = -3$

NOT possible

$-1 \le \cos x \le +1$



Ans $70.5°$ and $289.5°$

Find all values for x in the interval $0 \leq x < 360°$, for which

$$2\cos^2 x - 3\sin^2 x = 14\cos x$$

change this!

$2\cos^2 x - 3(1-\cos^2 x) = 14\cos x$

$2\cos^2 x - 3 + 3\cos^2 x = 14\cos x$

$5\cos^2 x - 14\cos x - 3 = 0$

Quadratic

$5m^2 - 14m - 3 = 0$

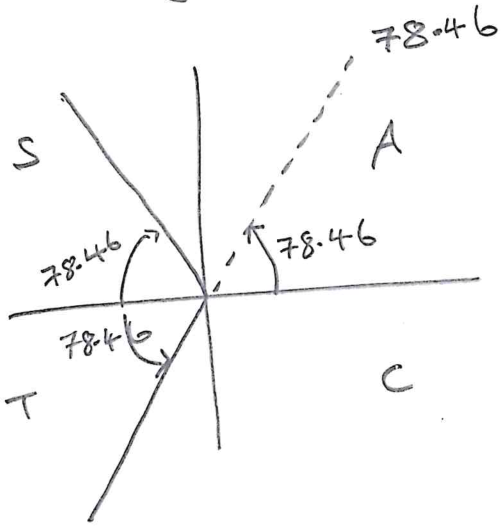
$(5m+1)(m-3) = 0$

$(5\cos x + 1)(\cos x - 3) = 0$ Each ( ) = 0

$\cos x = -1/5$

$\cos x = 3$
Not possible

$\cos x$ must be
$-1 \leq \cos x \leq +1$



$\cos^{-1}(1/5) = 78.46$

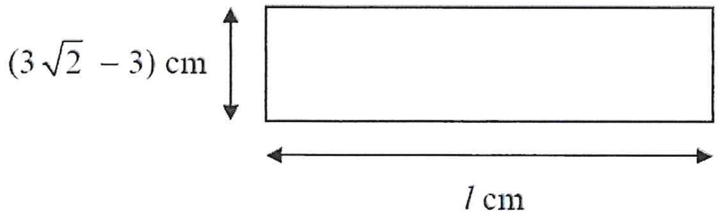
In the S&T quadrants

$180 - 78.46 = 101.54$

$180 + 78.46 = 258.46$

Ans $x = 101.54$ & 258.46

✓



The diagram shows a rectangle measuring $(3\sqrt{2} - 3)$ cm by l cm.
Given that the area of the rectangle is 6 cm^2, find the exact value of l in its simplest form.

$$(3\sqrt{2} - 3) \times l = 6$$

$$l = \frac{6}{(3\sqrt{2} - 3)}$$

Rationalise the denominator

$$l = \frac{6}{(3\sqrt{2} - 3)} \times \frac{(3\sqrt{2} + 3)}{(3\sqrt{2} + 3)}$$

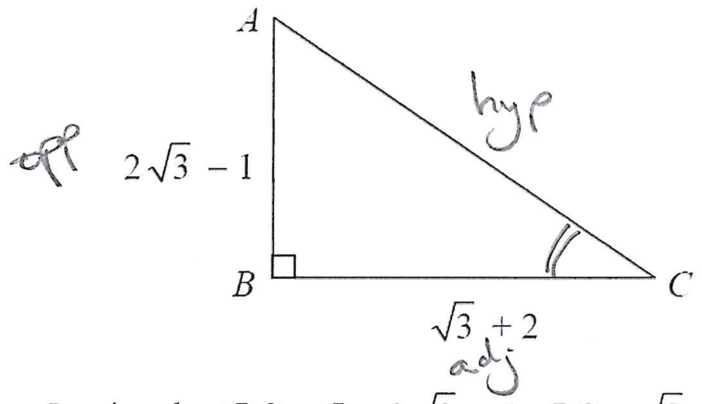
FOIL on denominator

$$l = \frac{18\sqrt{2} + 18}{18 + \cancel{9\sqrt{2}} - \cancel{9\sqrt{2}} - 9}$$

$$l = \frac{18\sqrt{2} + 18}{9}$$

$$l = 2\sqrt{2} + 2$$

5



In triangle ABC, $AB = 2\sqrt{3} - 1$, $BC = \sqrt{3} + 2$ and $\angle ABC = 90°$.

a Find the exact area of triangle ABC in its simplest form.

b Show that $AC = 2\sqrt{5}$.

c Show that $\tan(\angle ACB) = 5\sqrt{3} - 8$.

(a) Area $= \dfrac{(\sqrt{3}+2)(2\sqrt{3}-1)}{2} = \dfrac{2\times 3 - \sqrt{3} + 4\sqrt{3} - 2}{2}$

$= \dfrac{4 + 3\sqrt{3}}{2}$

Answer $= \underline{\underline{2 + \frac{3}{2}\sqrt{3}}}$

(b) $AC^2 = AB^2 + BC^2$

$AC^2 = (2\sqrt{3}-1)^2 + (\sqrt{3}+2)^2$

$= (2\sqrt{3}-1)(2\sqrt{3}-1) + (\sqrt{3}+2)(\sqrt{3}+2)$

$= 13 - 4\sqrt{3} + 7 + 4\sqrt{3}$

$= 20$

$AC = \sqrt{20} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$

(c) $\tan(A\hat{C}B) = \dfrac{2\sqrt{3}-1}{\sqrt{3}+2}$ Rationalise the denominator

$\dfrac{(2\sqrt{3}-1)}{(\sqrt{3}+2)} \times \dfrac{(\sqrt{3}-2)}{(\sqrt{3}-2)} = \dfrac{6 - 4\sqrt{3} - \sqrt{3} + 2}{3 - 4} = \dfrac{8 - 5\sqrt{3}}{-1}$

$= 5\sqrt{3} - 8$

(a) Show that the equation

$$3\sin 2x \tan 2x = \cos 2x + 2$$

Can be written in the form

$$4\cos^2 2x + 2\cos 2x - 3 = 0$$

(4)

(b) Find all values for x in the interval $0 \leq x < 180°$, for which

$$3\sin 2x \tan 2x = \cos 2x + 2$$

Give your answers to two decimal places.

(6)

(a) $3\sin 2x . \dfrac{\sin 2x}{\cos 2x} = \cos 2x + 2$

multiply everyTerm by $\cos 2x$

$3\sin 2x . \sin 2x = \cos^2 2x + 2\cos 2x$

$\underline{3\sin^2(2x)} = \cos^2(2x) + 2\cos(2x)$

$3(1 - \cos^2(2x)) = \cos^2(2x) + 2\cos(2x)$

$3 - 3\cos^2(2x) = \cos^2(2x) + 2\cos(2x)$

$0 = 4\cos^2(2x) + 2\cos(2x) - 3$ ✓

(b) Solve $4\cos^2(2x) + 2\cos(2x) - 3 = 0$ quadratic

$4m^2 + 2m - 3 = 0$

$am^2 + bm + c$

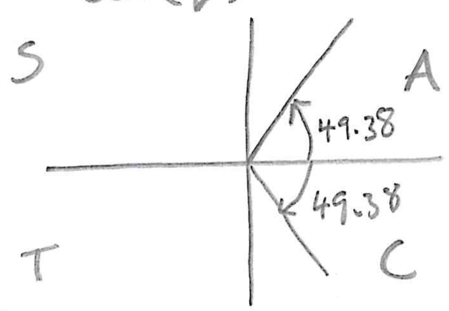
$\cos(2x) = \dfrac{-2 \pm \sqrt{2^2 - 4(4)(-3)}}{2 \times 4}$

$\cos(2x) = \dfrac{-2 \pm \sqrt{52}}{8}$ $\qquad \cos 2x = 0.651$ & -1.151

not possible

only $\cos 2x = 0.651$

$\cos(q) = 0.651$



$q = 49.38$ & 310.62

$2x = 49.38$ & 310.62

$x = 24.69$ ✓ & 155.3 ✓

Prove that

a $(\sin x + \cos x)^2 \equiv 1 + 2\sin x \cos x$

b $\frac{1}{\cos x} - \cos x \equiv \sin x \tan x, \quad \cos x \neq 0$

LHS

$(\sin x + \cos x)^2$

$(\sin x + \cos x)(\sin x + \cos x)$

F O I L

$\sin^2 x + \sin x \cos x + \cos x \sin x + \cos^2 x$

$\underbrace{\sin^2 x + \cos^2 x}$ $+ 2\sin x \cos x$

$1 + 2\sin x \cos x$ ✓

(b) LHS $\frac{1}{\cos x} - \cos x$

common denominator

$\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$

$\frac{1 - \cos^2 x}{\cos x}$

$\frac{\sin^2 x}{\cos x}$ Because $\sin^2 + \cos^2 = 1$
$\sin^2 = 1 - \cos^2$

$\sin x \times \left(\frac{\sin x}{\cos x}\right)$

$\sin x \times \tan x$ ✓

4

Given that the point with coordinates $(1+\sqrt{3}, 5\sqrt{3})$ lies on the curve with the equation

$$y = 2x^2 + px + q,$$

find the values of the rational constants p and q.

$y = 2x^2 + px + q$

has point $(1+\sqrt{3}, 5\sqrt{3})$

Substitute in the point into the curve

$5\sqrt{3} = 2(1+\sqrt{3})^2 + p(1+\sqrt{3}) + q$

$5\sqrt{3} = 2(1+\sqrt{3})(1+\sqrt{3}) + p + p\sqrt{3} + q$

$5\sqrt{3} = 2(1+2\sqrt{3}+3) + p + p\sqrt{3} + q$

$5\sqrt{3} = 8+4\sqrt{3} + p + p\sqrt{3} + q$

$5\sqrt{3} = 8+p+q + (4+p)\sqrt{3}$

But $5\sqrt{3}$ must be $(4+p)\sqrt{3}$

so $5 = 4+p$

Ans $p = 1$

Then $0 = 8+p+q$

$0 = 8+1+q$

$0 = 9+q$

Ans $q = -9$

a Simplify $(2-\sqrt{3})(2+\sqrt{3})$.

b Express $\dfrac{2}{2-\sqrt{3}}$ in the form $a+b\sqrt{3}$.

$(2-\sqrt{3})(2+\sqrt{3})$

F O I L

$4 + 2\sqrt{3} - 2\sqrt{3} - 3$

1

check on your calculator

Ans 1

$$\frac{2}{2-\sqrt{3}} = \frac{2}{(2-\sqrt{3})} \times \frac{2+\sqrt{3}}{(2+\sqrt{3})} = \frac{4+2\sqrt{3}}{1}$$

$$= 4+2\sqrt{3}$$

Denominator

$(2-\sqrt{3})(2+\sqrt{3})$

is part a of the question.

4