a Prove the identity

$$(\cos x - \tan x)^2 + (\sin x + 1)^2 \equiv 2 + \tan^2 x.$$

Hence, find the values of x in the interval $0 \leq x \leq 360°$ for which

$$(\cos x - \tan x)^2 + (\sin x + 1)^2 = 3.$$

LHS

$$(\cos x - \tan x)^2 + (\sin x + 1)^2$$

$$(\cos x - \tan x)(\cos x - \tan x) + (\sin x + 1)(\sin x + 1)$$

$$\cos^2 x - 2\cos x \tan x + \tan^2 x + \sin^2 x + 2\sin x + 1$$

$$\underbrace{\cos^2 x + \sin^2 x} - 2\cos x \tan x + 2\sin x + \tan^2 x + 1$$

$$1 - 2\cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}} + 2\sin x + \tan^2 x + 1$$

$$1 - \cancel{2\sin x} + \cancel{2\sin x} + \tan^2 x + 1$$

$$1 + \tan^2 x + 1$$

$$2 + \tan^2 x$$

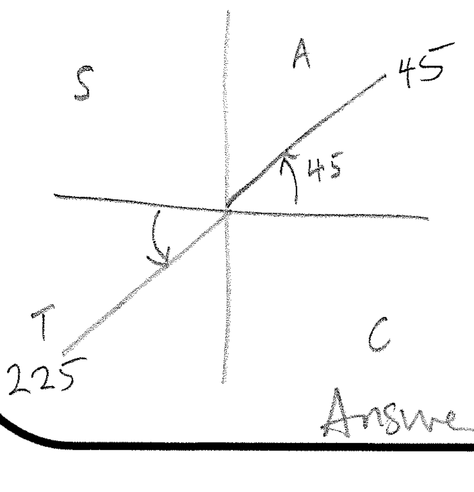
Solve

$$2 + \tan^2 x = 3$$

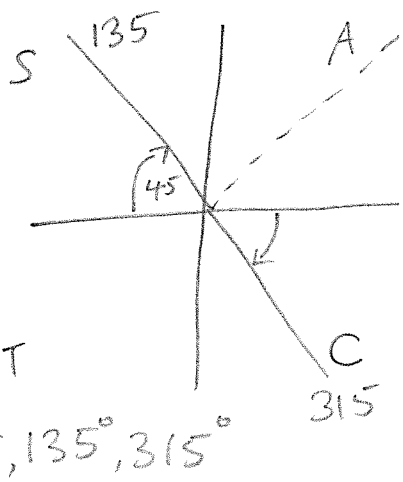
$$\tan^2 x = 1$$

$$\tan x = \pm 1$$

$\tan x = +1$



$\tan x = -1$



Answer $45°, 225°, 135°, 315°$

Prove that

$$\frac{\cos^2 x}{1-\sin x} \equiv 1+\sin x, \quad \sin x \neq 1$$

$$\frac{\cos^2 x}{1-\sin x}$$

$$\frac{1-\sin^2 x}{1-\sin x}$$ DOTS Difference of 2 squares

$$\frac{(1+\sin x)\cancel{(1-\sin x)}}{\cancel{1-\sin x}}$$

$$(1+\sin x)$$

$$\frac{1+\sin x}{\cos x} \equiv \frac{\cos x}{1-\sin x}, \quad \cos x \neq 0$$

LHS $\frac{1+\sin x}{\cos x}$

$$\frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

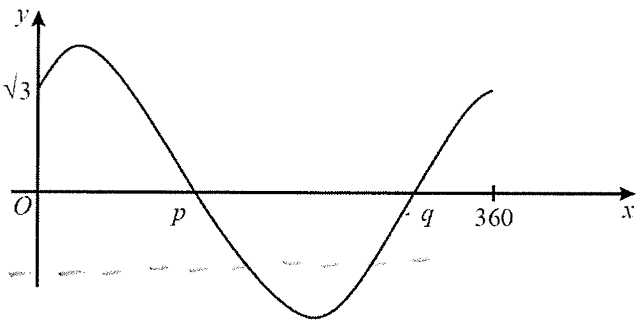
$$\frac{\cos x}{\cos^2 x} + \frac{\sin x \cos x}{\cos^2 x}$$

$$\frac{\cos x + \sin x \cos x}{\cos^2 x}$$

$$\frac{\cos x\,(1+\sin x)}{1-\sin^2 x}$$

$$\frac{\cos x\,\cancel{(1+\sin x)}}{\cancel{(1+\sin x)}(1-\sin x)}$$

$$\frac{\cos x}{1-\sin x}$$



The diagram above shows the curve with equation $y = k \sin (x + 60)°$, $0 \leq x \leq 360$, where k is a constant.

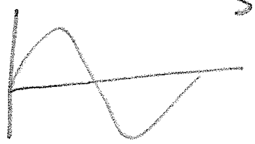
The curve meets the y-axis at $(0, \sqrt{3})$ and passes through the points $(p, 0)$ and $(q, 0)$.

(a) Show that $k = 2$.

(1)

(b) Write down the value of p and the value of q.

Sin curve



(2)

The line $y = -1.6$ meets the curve at the points A and B.

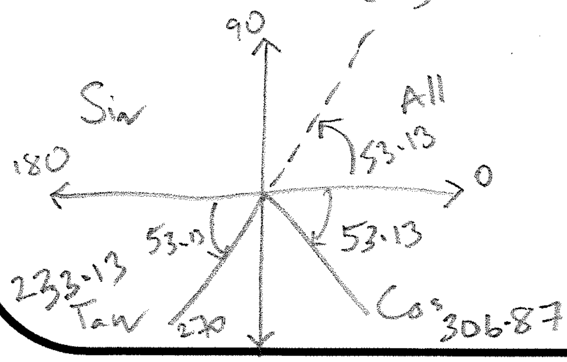
(c) Find the x-coordinates of A and B, giving your answers to 1 decimal place.

(5)

(a)
$y = k\sin(x + 60)$
Put in $x = 0$ and $y = \sqrt{3}$
$\sqrt{3} = k\sin(60)$
$\sqrt{3} = k \cdot \frac{\sqrt{3}}{2}$
$\frac{2\sqrt{3}}{\sqrt{3}} = k$
$2 = k$

(b) solve
$0 = 2\sin(x+60)$
$0 = \sin(x+60)$
$x + 60 = 180$ $\quad$ $x+60 = 360$
$x = 120$ $\quad$ $x = 300$
check these!
Answer $x = 120°$ & $300°$

(c) $-1.6 = 2\sin(x+60)$
$-0.8 = \sin(x+60)$
$-0.8 = \sin(m)$



$m = 233.13$ & 306.87
$x + 60 = 233.13$ & 306.87
$x = 173.13$ & 246.87
Answers

Check it

(a) Given that $\sin\theta = 5\cos\theta$, find the value of $\tan\theta$.

(1)

(b) Hence, or otherwise, find the values of θ in the interval $0 \leq \theta < 360°$ for which

$$\sin\theta = 5\cos\theta,$$

giving your answers to 1 decimal place.

(3)

(a)

$\sin\theta = 5\cos\theta$

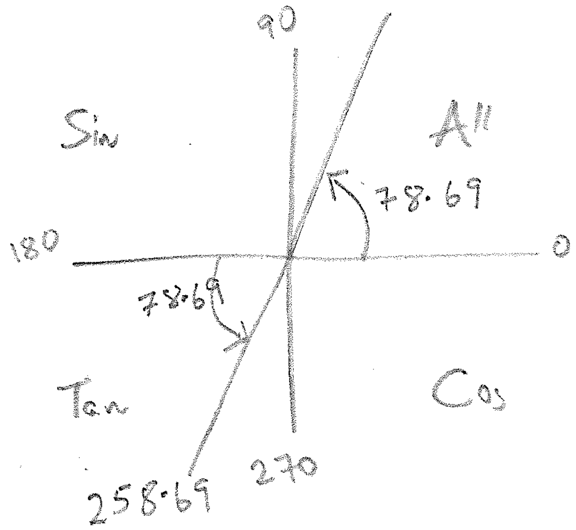
$\frac{\sin\theta}{\cos\theta} = 5$

$\tan\theta = 5$

(b) Solve $\tan\theta = 5$ for $0 \leq \theta < 360$

$\theta = \tan^{-1}(5)$

$\theta = 78.69°$



Answer
78.69 & 258.69

Answer
78.7° & 259°

(3 sig. fig)

(i) Solve, for $-180° \leq \theta < 180°$,

$$(1 + \tan\theta)(5\sin\theta - 2) = 0.$$

(4)

(ii) Solve, for $0 \leq x < 360°$,

$$4\sin x = 3\tan x.$$

(6)

(i) $(1+\tan\theta)(5\sin\theta - 2) = 0$

each ( ) = 0

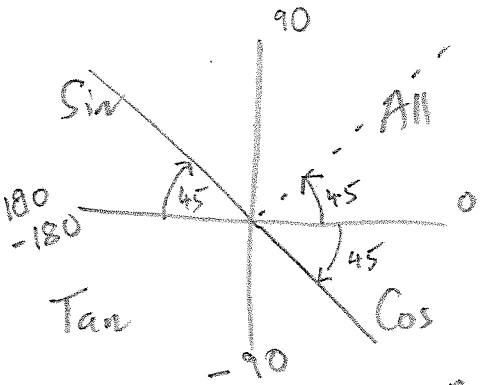
$1 + \tan\theta = 0$

$\tan\theta = -1$

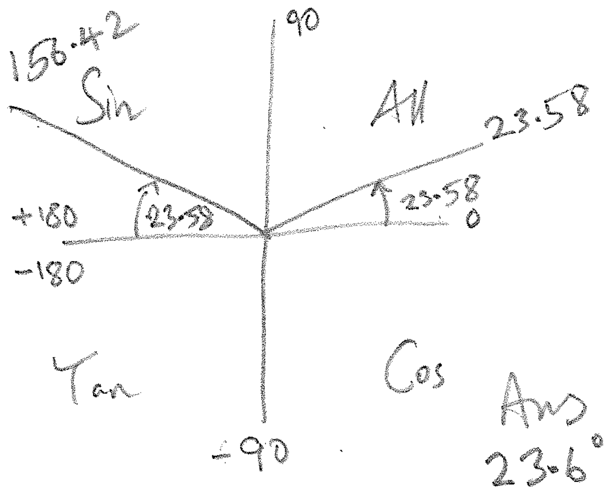
$5\sin\theta - 2 = 0$

$\sin\theta = \frac{2}{5}$

$\sin^{-1}(2/5)$



Ans $\underline{135°}$ & $\underline{-45°}$



Ans $\underline{23.6°}$ & $\underline{156}$

(ii) $4\sin x = 3\tan x$

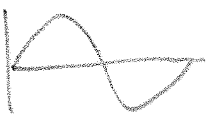
$4\sin x = 3\frac{\sin x}{\cos x}$

$4\sin x\cos x = 3\sin x$

$4\sin x\cos x - 3\sin x = 0$

$(\sin x)(4\cos x - 3) = 0$

Each ( ) = 0

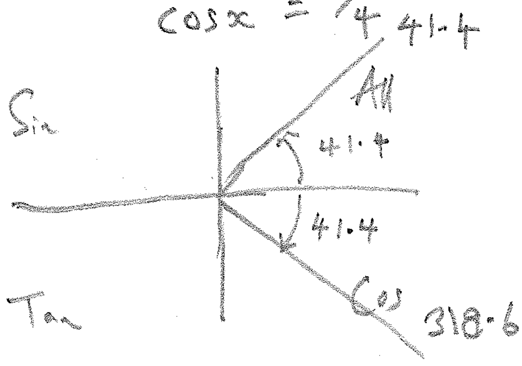


$\sin x = 0$

Answer $x = 0°, 180°, 360°$

$4\cos x - 3 = 0$

$\cos x = 3/4$



Answer $41.4°$ & $319°$

(a) Solve, for $-180° \le x < 180°$, the equation

$$3\sin^2 x + \sin x + 8 = 9\cos^2 x$$

giving your answers to 2 decimal places.

What links $\sin^2\theta + \cos^2\theta = 1$

(6)

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9\cos^2(2\theta - 30°)$$

giving your answer to 2 decimal places.

(2)

$3\sin^2 x + \sin x + 8 = 9\cos^2 x$

$3\sin^2 x + \sin x + 8 = 9(1 - \sin^2 x)$

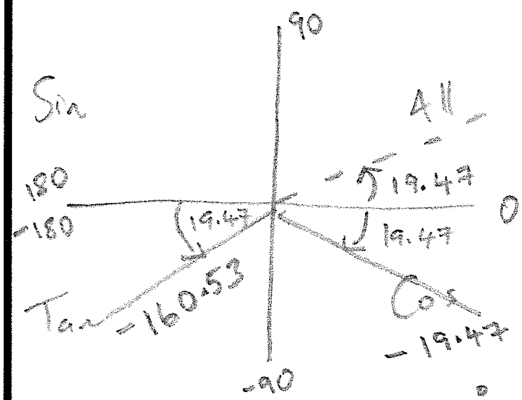
$12\sin^2 x + \sin x - 1 = 0$

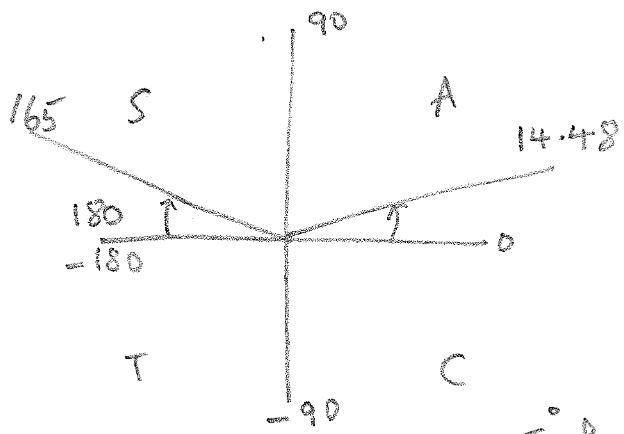
$(3\sin x + 1)(4\sin x - 1) = 0$

Each ( ) = 0

$3\sin x + 1 = 0$ $\qquad$ $4\sin x - 1 = 0$

$\sin x = -1/3$ $\qquad$ $\sin x = 1/4$





Ans $-19.5°$ & $-161°$ $\qquad$ Ans $14.5°$ & $165°$

Hence

$2\theta - 30 = -19.5$ $\qquad$ $2\theta - 30 = 14.5$

$2\theta = 10.53$ $\qquad$ $2\theta = 44.5$

$\theta = 5.27$ $\qquad$ $\theta = 22.25$

(a) Show that the equation

$$4 \cos \theta - 1 = 2 \sin \theta \tan \theta$$

can be written in the form

$$6 \cos^2\theta - \cos \theta - 2 = 0$$

(4)

(b) Hence solve, for $0 \leq x < 90°$

$$4 \cos 3x - 1 = 2 \sin 3x \tan 3x$$

giving your answers, where appropriate, to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(a) $4\cos\theta - 1 = 2\sin\theta \cdot \dfrac{\sin\theta}{\cos\theta}$

Replace $\sin^2\theta = 1 - \cos^2\theta$

$4\cos^2\theta - \cos\theta = 2\sin^2\theta$

$4\cos^2\theta - \cos\theta = 2(1 - \cos^2\theta)$

$4\cos^2\theta - \cos\theta = 2 - 2\cos^2\theta$

$6\cos^2\theta - \cos\theta - 2 = 0$ ✓

(b) $4\cos(3x) - 1 = 2\sin(3x)\tan(3x)$

Hence is an important word

from part (a)

$6\cos^2(3x) - \cos(3x) - 2 = 0$

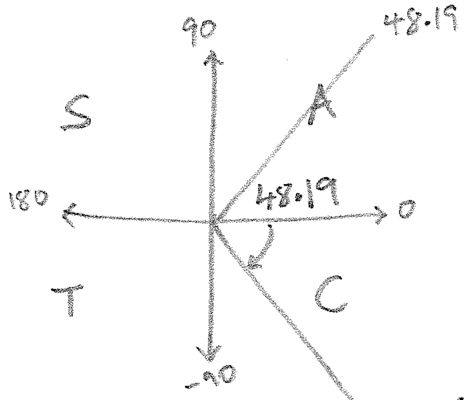
$(3\cos(3x) - 2)(2\cos(3x) + 1) = 0$

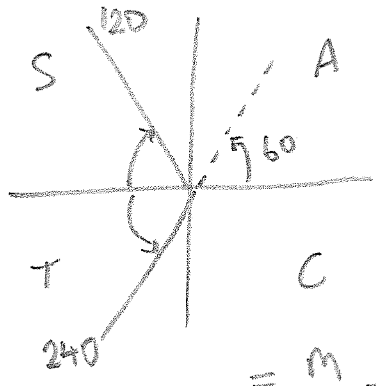
$\cos(3x) = \frac{2}{3}$ $\qquad$ $\cos(3x) = -\frac{1}{2}$

Kylie Question!

$\cos(m) = \frac{2}{3}$ $\qquad$ $\cos(m) = -\frac{1}{2}$

$3x$ means





$m = 120°$ & $240°$

$m = 48.19, 311.81°$

$x = \dfrac{m}{3} = 16.1°, 40°, 80°$ ✓
check

Solve, for $360° \le x < 540°$, strange interval

$$12\sin^2 x + 7\cos x - 13 = 0$$

Give your answers to one decimal place.

$12\sin^2 x + 7\cos x - 13 = 0$

$12(1-\cos^2 x) + 7\cos x - 13 = 0$

$12 - 12\cos^2 x + 7\cos x - 13 = 0$

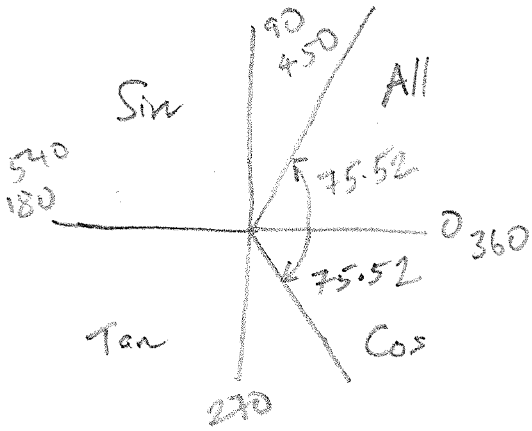
$0 = 12\cos^2 x - 7\cos x + 1$

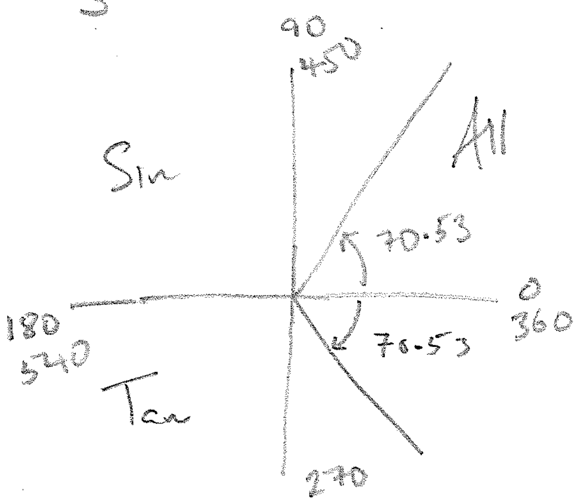
Now quadratic. Factorise it $(\quad)(\quad) = 0$

$12\cos^2 x - 7\cos x + 1 = 0$

$(4\cos x - 1)(3\cos x - 1) = 0$ Kylie!

$\cos x = 1/4$ $\cos x = 1/3$





$x = \ldots, 435.52°, \ldots$

$x = \ldots, 430.53°, \ldots$

Answer $\underline{435.5°}$, ... , $\underline{430.5°}$, ...

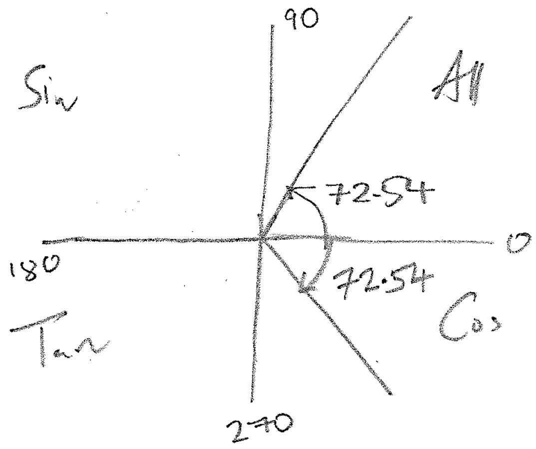
Solve, for $0 \leq x < 180°$, the equation, Kylie Question

$$\cos(2x + 15) = 0.3$$

Give your answers to one decimal place.

I don't like solving $\cos(2x+15)=0.3$
so just solve $\cos(m)=0.3$ then adjust

$\cos(m)=0.3$



$m = 72.54°$

$m = 287.46°$

$m = 360 + 72.54$
but outside our interval range.

$m = 72.54$	$m = 287.46$
$2x+15 = 72.54$	$2x+15 = 287.46$
$2x = 57.54$	$2x = 272.46$
$x = 28.77$	$x = 136.23$

Ans $x = 28.8°$ & $136.2°$

check on your new calculator! ✓

(a) Show that

$$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} \equiv 4 - 5\cos\theta$$

(4)

(b) Hence, or otherwise, solve, for $0 \leq x \leq 360°$, the equation

$$\frac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 3\sin x$$

(3)

show $\dfrac{10\sin^2\theta - 7\cos\theta + 2}{3+2\cos\theta}$

$\dfrac{10(1-\cos^2\theta) - 7\cos\theta + 2}{3+2\cos\theta}$

$\dfrac{10 - 10\cos^2\theta - 7\cos\theta + 2}{3+2\cos\theta}$

$\dfrac{12 - 7\cos\theta - 10\cos^2\theta}{3+2\cos\theta}$

$\dfrac{(3+2\cos\theta)(4-5\cos\theta)}{(3+2\cos\theta)}$

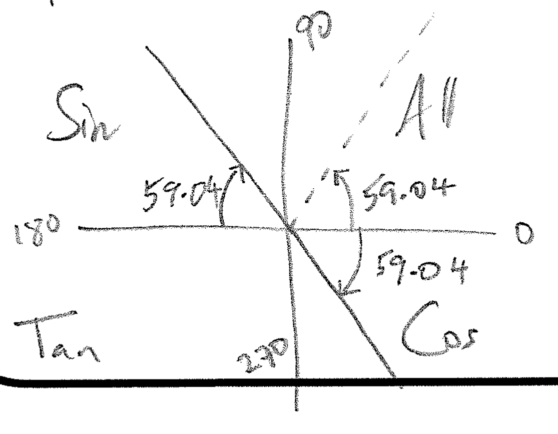
$4 - 5\cos\theta$ ✓

Solve $4 - 5\cos x = 4 + 3\sin x$

$-5\cos x = 3\sin x$

$\dfrac{-5}{3} = \tan x$

$\tan x = -5/3$



Ans $120.96°$ & $300.96°$

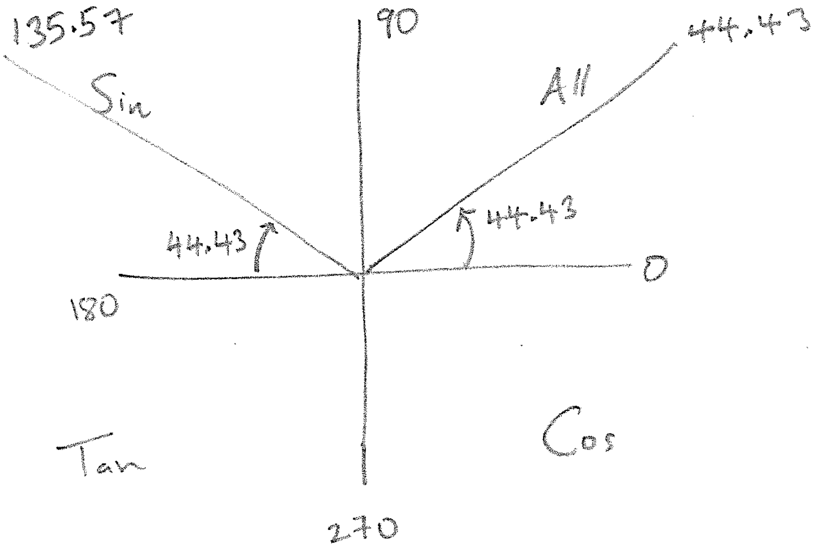
Solve, for $0 \leq \theta < 180°$, the equation, Kylie Question

$$\sin(3\theta - 15) = 0.7$$

Give your answers to two decimal places.

$\sin(3\theta - 15) = 0.7$

$\sin(m) = 0.7$ go "spinning around" 3 times so between $0 \leq \theta < 540$



m values $m = 44.43, 135.57, 404.43, 490.57$

$3m - 15 = m = 44.43, 135.57, 404.43, 490.57$

$3m - 15 = 44.43$

$3m = 59.43$

$m = 19.81$

$3m - 15 = 135.57$

$3m = 150.57$

$m = 50.19$

$3m - 15 = 404.43$

$3m = 419.43$

$m = 139.81$

$3m - 15 = 490.57$

$3m = 505.57$

$m = 168.52$

Ans 19.81, 50.19, 139.81 & 168.52

check these!

(a) Show that the equation

$$3\sin 2x \tan 2x = \cos 2x + 2$$

Can be written in the form

$$4\cos^2 2x + 2\cos 2x - 3 = 0$$

(4)

(b) Find all values for x in the interval $0 \le x < 180°$, for which

$$3\sin 2x \tan 2x = \cos 2x + 2$$

Give your answers to two decimal places.

(6)

$$3\sin 2x \tan 2x = \cos 2x + 2$$

$$3\sin 2x \frac{\sin 2x}{\cos 2x} = \cos 2x + 2$$

$$3\sin^2(2x) = \cos^2(2x) + 2\cos(2x)$$

$$3(1 - \cos^2(2x)) = \cos^2(2x) + 2\cos(2x)$$

$$3 - 3\cos^2(2x) = \cos^2(2x) + 2\cos(2x)$$

$$0 = 4\cos^2(2x) + 2\cos(2x) - 3$$

solve by Factorising $(\quad)(\quad) = 0$

$$0 = (4\cos(2x) \quad)(\cos(2x) \quad)$$

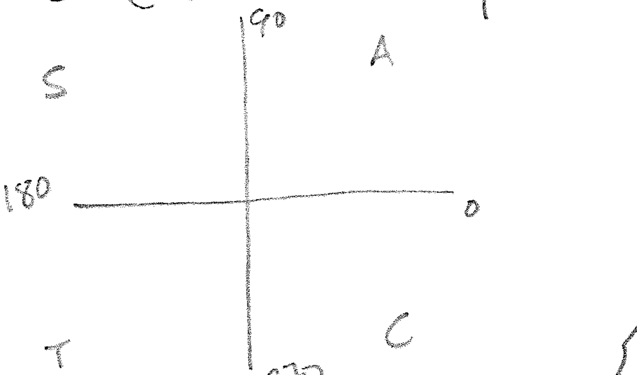
Doesn't factorise. So Quadratic Formula

or new calculator

$$\cos(2x) = \frac{-1-\sqrt{13}}{4} \quad \& \quad \frac{-1+\sqrt{13}}{4}$$

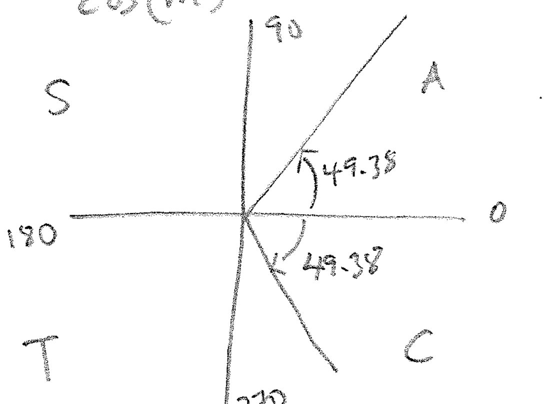
$\cos(2x) = -1.151$

$\cos(m) = -1.151$ Not possible



$\cos(2x) = +0.651$

$\cos(m) = 49.38$



Ans $24.69°$, $155.31°$