Given that $\mathbf{r} = 6\mathbf{i} + c\mathbf{j}$, where c is a positive constant, find the value of c such that

a $\mathbf{r}$ is parallel to the vector $2\mathbf{i} + \mathbf{j}$

b $\mathbf{r}$ is parallel to the vector $-9\mathbf{i} - 6\mathbf{j}$

c $|\mathbf{r}| = 10$

d $|\mathbf{r}| = 3\sqrt{5}$

(a) parallel means scalar multiple

$6i + cj = \lambda(2i + j)$

$\times 3$

λ must be 3

Ans $c = 3$

(b) $-9i - 6j = \lambda(6i + cj)$

$\lambda = -1.5$

$-9i - 6j = -1.5(6i + cj)$

Ans $c = 4$

(c) $|r| = 10$

$|r| =$ magnitude
modulus of r

$|6i + cj| = 10$

$\sqrt{6^2 + c^2} = 10$

Pythagoras'

$6^2 + c^2 = 10^2$

$36 + c^2 = 100$

$c^2 = 64$

$c = \sqrt{64}$

Ans $c = 8$ (must be positive)

(d) $|r| = 3\sqrt{5}$

$|6i + cj| = 3\sqrt{5}$

$\sqrt{6^2 + c^2} = 3\sqrt{5}$

$6^2 + c^2 = (3\sqrt{5})^2$

$6^2 + c^2 = 45$

$c^2 = 45 - 36$

$c^2 = 9$

$c = \pm 3$

Ans $c = +3$

must be positive

Given that $|3\mathbf{i} + k\mathbf{j}| = 3\sqrt{17}$

Find the value of k

$|3i + kj|$ is magnitude of vector

$$\sqrt{3^2 + k^2} = 3\sqrt{17}$$

$$9 + k^2 = (3\sqrt{17})^2$$

$$k^2 = 153 - 9$$

$$k^2 = 144$$

$$k = \pm 12$$

Ans $k = +12$ or -12

$3i + 12j$ or $3i - 12j$

You could check

$\sqrt{3^2 + 12^2}$ $\qquad$ $\sqrt{3^2 + (-12)^2}$

$\sqrt{9 + 144}$ $\qquad$ $\sqrt{9 + 144}$

$\sqrt{153}$ $\qquad$ $\sqrt{153}$

The points D, E and F have coordinates (-3, 2), (4, -1) and (1, -8) respectively.

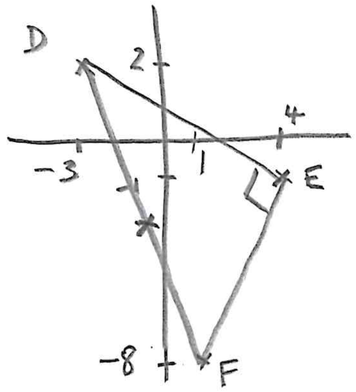
(a) Show that angle DEF is a right angle. $90°$ check gradients **(4)**

Given that D, E and F all lie on the circle C.

(b) Find the coordinates of the centre of C. **(3)**

(c) Find the equation of the circle C. **(3)**

Sketch out co-ordinates



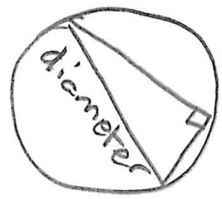
Gradient DE $= \frac{-3}{7}$

Gradient EF $= \frac{7}{3}$

Gradient of perpendicular lines multiply to -1

$\frac{-3}{7} \times \frac{7}{3} = -1$

DF must be diameter



angle in semi-circle is $90°$

Find mid-point of DF

centre $(-1, -3)$

$M = (-1, -3)$

Find radius of circle Distance from M to F

Pythagoras

$r^2 = 2^2 + 5^2$

$r^2 = 4 + 25$

$r^2 = 29$

Equation

$(x - \ \ )^2 + (y - \ \ )^2 = r^2$ format

$(x + 1)^2 + (y + 3)^2 = 29$

centre $(-1, -3)$ radius $\sqrt{29}$

The circle C has the equation $x^2 + y^2 - 6x + 2y = 6$

circle format

$(x- \ )^2 + (y- \ )^2 = r^2$

(a) Find the coordinates of the centre and the radius of C (3)

C crosses the y axis at the points A and B

(b) Find the coordinates of the points A and B (3)

$x^2 + y^2 - 6x + 2y = 6$

Complete the square

$x^2 - 6x \quad + y^2 + 2y = 6$

$(x-3)^2 - 9 + (y+1)^2 - 1 = 6$

$(x-3)^2 + (y+1)^2 = 16 \quad = 4^2$

Centre $(3, 1)$ radius $= 4$

Crosses y axis at A & B

crosses y axis when $x = 0$

Put in $x = 0$

$x^2 + y^2 - 6x + 2y = 6$

$0^2 + y^2 - 0 + 2y = 6$

$y^2 + 2y - 6 = 0$

use equation solver or $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = -1 + \sqrt{7}$ $\quad$ $y = -1 - \sqrt{7}$

Ans $A(0, -1 + \sqrt{7})$

$B(0, -1 - \sqrt{7})$

Given that $\mathbf{p} = \mathbf{i} + 3\mathbf{j}$ and $\mathbf{q} = 4\mathbf{i} - 2\mathbf{j}$,

a find the values of a and b such that $a\mathbf{p} + b\mathbf{q} = -5\mathbf{i} + 13\mathbf{j}$,

b find the value of c such that $c\mathbf{p} + \mathbf{q}$ is parallel to the vector $\mathbf{j}$,

c find the value of d such that $\mathbf{p} + d\mathbf{q}$ is parallel to the vector $3\mathbf{i} - \mathbf{j}$.

$a(i+3j) + b(4i-2j) = -5i+13j$

$\begin{pmatrix} a \\ 3a \end{pmatrix} + \begin{pmatrix} 4b \\ -2b \end{pmatrix} = \begin{pmatrix} -5 \\ +13 \end{pmatrix}$ $\quad a + 4b = -5$

$3a - 2b = 13$

solve simultaneously

Ans $a = 2 \quad b = -7/2$

$cp + q$ is parallel to j

$cp + q$ has no i component in vector

$c\begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ \end{pmatrix}$

Look at only i component

$c + 4 = 0$ $\quad$ Ans $c = -4$

$p + dq$ is parallel to $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

parallel means scalar multiple

$\begin{pmatrix} 1 \\ 3 \end{pmatrix} + d\begin{pmatrix} 4 \\ -2 \end{pmatrix} = \lambda\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$1 + 4d = 3\lambda \qquad 4d - 3\lambda = -1$

$3 - 2d = -\lambda \qquad 2d - \lambda = 3$

Solve simultaneously

$d = 5$

$\lambda = 7$ $\qquad$ Ans $d = 5$

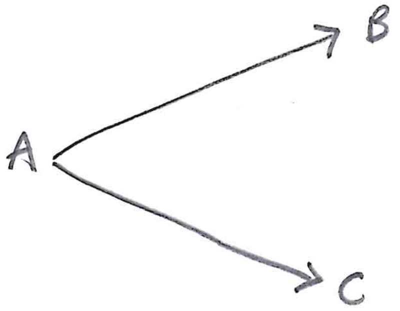
check

$\begin{pmatrix} 1 \\ 3 \end{pmatrix} + 5\begin{pmatrix} 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 21 \\ -7 \end{pmatrix}$ which is parallel to $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

In triangle ABC. $\overrightarrow{AB} = 6\mathbf{i} + 2\mathbf{j}$. $\overrightarrow{AC} = 8\mathbf{i} - 5\mathbf{j}$

(a) Find the vector $\overrightarrow{BC}$ **(2)**

(b) Find the length of the line AB **(2)**



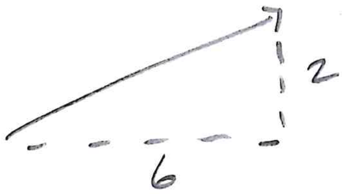
$\overrightarrow{AB} = 6i + 2j$

$\overrightarrow{AC} = 8i - 5j$

$$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$$
$$= (-6i - 2j) + (8i - 5j)$$
$$= 2i - 7j$$

$|\overrightarrow{AB}| = |$magnitude of $AB|$

length always think Pythagoras'



$$\overrightarrow{AB} = 6i + 2j$$
$$|\overrightarrow{AB}| = \sqrt{6^2 + 2^2}$$
$$|\overrightarrow{AB}| = \sqrt{36 + 4} = \sqrt{40}$$

circle $(x- \ )^2+(y- \ )^2=r^2$

The circle C has centre (1, 5) and passes through the point A (-4, 3).

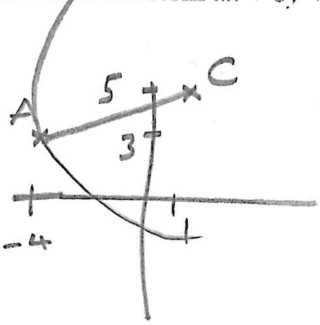
(a) Find an equation for C.

(4)

(b) Find an equation for the tangent to C and A, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers

(4)

$(x-1)^2+(y-5)^2=r^2$



Distance CA is radius.

$r = \sqrt{5^2+2^2}$

Pythagoras

$r = \sqrt{25+4}$

$r = \sqrt{29}$

Ans $(x-1)^2+(y-5)^2 = 29$

Tangent at A = ? $y = mx+c$

Find gradient of radius AC $= 2/5$

then gradient of tangent at A $= -5/2$

because it is perpendicular

$y = -5/2\, x + c$

Substitute in point A $(-4,3)$

$3 = -5/2(-4) + c$

$3 = 10 + c$

$-7 = c$

$y = -5/2\, x - 7$

$2y = -5x - 14$

$5x+2y+14=0$

Ans $5x+2y+14=0$

$(x- \ )^2+(y - \ )^2 = r^2$ circle format

The point P lies on the circle with equation $x^2 + y^2 + 12x - 6y + 27 = 0$ and the point Q has coordinates (8, 1). Find the minimum length of PQ giving your answer in the form $k\sqrt{2}$.

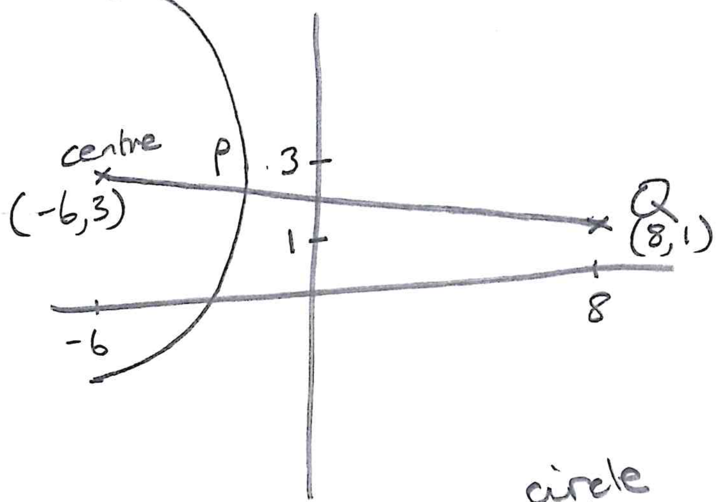
$x^2 + 12x + y^2 - 6y + 27 = 0$ complete the square

$(x+6)^2 - 36 + (y-3)^2 - 9 = -27$

$(x+6)^2 + (y-3)^2 = 45 - 27$

$(x+6)^2 + (y-3)^2 = 18$

$(x+6)^2 + (y-3)^2 = 18$



Distance between circle centre and Q

$= \sqrt{14^2 + 2^2}$

$= \sqrt{196 + 4}$

$= \sqrt{200}$

Radius $= \sqrt{18}$

Distance between P and Q $= \sqrt{200} - \sqrt{18}$

$\sqrt{200} - \sqrt{18}$

$10\sqrt{2} - 3\sqrt{2}$

$7\sqrt{2}$

Relative to a fixed origin O, the points A and B have position vectors $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$ respectively.

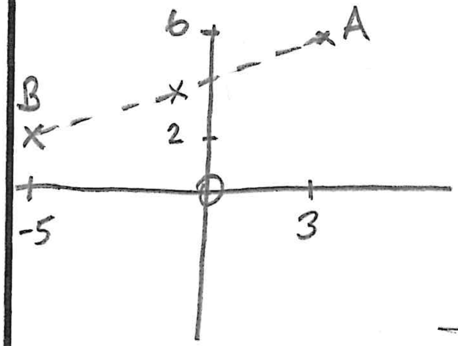
Find

a the vector $\overrightarrow{AB}$,

b $|\overrightarrow{AB}|$,

c the position vector of the mid-point of AB,

d the position vector of the point C such that $OABC$ is a parallelogram.



points A and B plotted

$\overrightarrow{AB} = \begin{pmatrix} -8 \\ -4 \end{pmatrix}$ $-8i - 4j$

Ans $\overrightarrow{AB} = -8i - 4j$

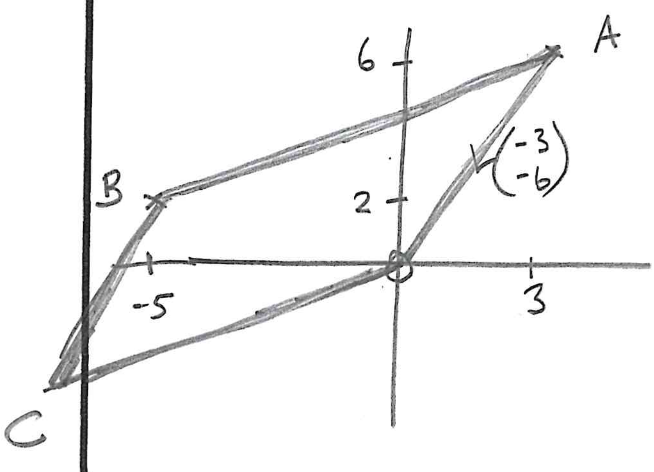
$|\overrightarrow{AB}| =$ magnitude of $AB = \sqrt{(-8)^2 + (-4)^2}$

$= \sqrt{64 + 16}$

$= \sqrt{80}$

Midpoint of AB

$\left(\frac{-5+3}{2}, \frac{2+6}{2} \right)$ Ans $(-1, 4)$



OABC you must go around in that order

$\overrightarrow{BC}$ must be same as $\overrightarrow{AO}$

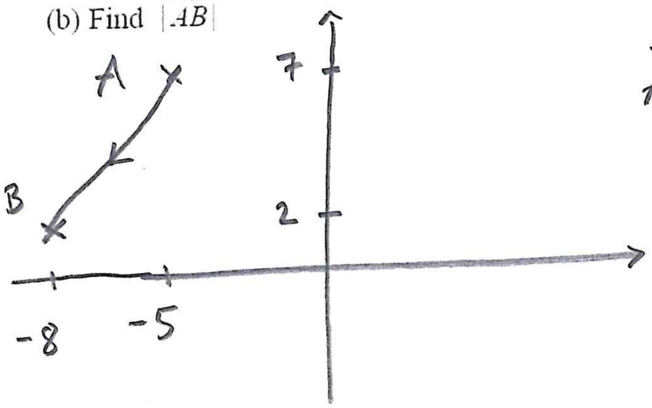
$\overrightarrow{BC} = \begin{pmatrix} -3 \\ -6 \end{pmatrix}$

Point C $(-8, -4)$

Given that the point A has position vector $-5\mathbf{i} + 7\mathbf{j}$ and the point B has position vector $-8\mathbf{i} + 2\mathbf{j}$

(a) Find the vector $\overrightarrow{AB}$ **(2)**

(b) Find $|\overrightarrow{AB}|$ **(2)**



$\overrightarrow{AB} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$

$\overrightarrow{AB} = -3i - 5j$

$|\overrightarrow{AB}|$ = magnitude of AB

Pythagoras'

$|\overrightarrow{AB}| = \sqrt{3^2 + 5^2}$

$= \sqrt{9 + 25}$

$= \sqrt{34}$

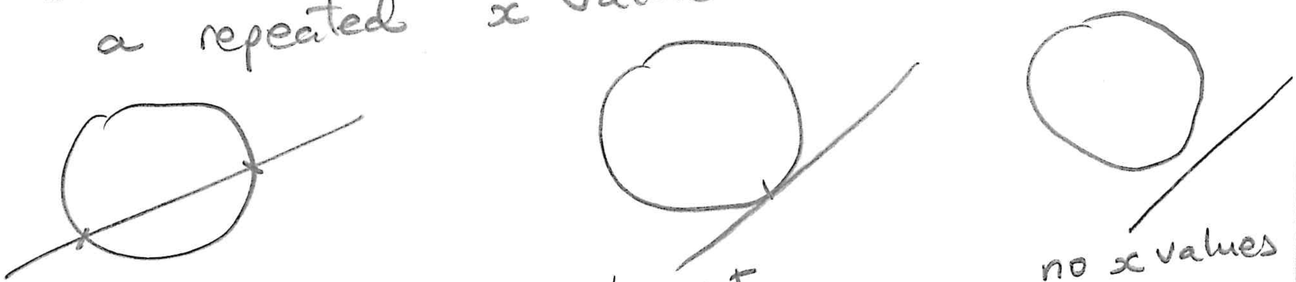
The line with equation $y = x + k$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 8y + 17 = 0$.
Find the two possible values of k.

tangent means $y = x+k$ meets circle at only one point

Substitute $y = x+k$ into circle

$$x^2 + y^2 + 6x - 8y + 17 = 0$$
$$x^2 + (x+k)^2 + 6x - 8(x+k) + 17 = 0$$
$$x^2 + x^2 + 2kx + k^2 - 6x - 8x - 8k + 17 = 0$$
$$2x^2 + (2k-14)x + (k^2 - 8k + 17) = 0$$

Quadratic but this must have a repeated x value



2 x values
not tangent
discriminent >0

tangent
one x value
discriminent $=0$

no x values
discriminent <0

$b^2 - 4ac$

$$(2k-14)^2 - 4(2)(k^2 - 8k + 17) = 0$$
$$4k^2 - 56k + 196 - 8k^2 + 64k - 136 = 0$$
$$-4k^2 + 8k + 60 = 0$$
$$4k^2 - 8k - 60 = 0$$
$$k^2 - 2k - 15 = 0$$
$$(k-5)(k+3) = 0$$

Ans $k = 5$
$k = -3$

touches circle just once

The line with equation $y = mx$ is a tangent to the circle with equation $x^2 + y^2 - 8x - 16y + 72 = 0$.

Find the two possible values of m.

Substitute $y = mx$ into circle

$x^2 + y^2 - 8x - 16y + 72 = 0$

$(x^2 + mx)^2 - 8x - 16(mx) + 72 = 0$

$x^2 + m^2x^2 - 8x - 16mx + 72 = 0$

$x^2(1 + m^2) + x(-8 - 16m) + 72 = 0$

Disciminent $= 0$

$b^2 - 4ac = 0$

$(-8 - 16m)^2 - 4(1 + m^2)(72) = 0$

$64 + 256m + 256m^2 - 288 - 288m^2 = 0$

$-32m^2 + 256m - 224 = 0$

$32m^2 - 256m + 224 = 0$

use equation solver on calculator

$32(m^2 - 8m + 7) = 0$

$32(m - 7)(m - 1) = 0$

$m = 7$ $m = 1$

Ans $m = 7$ & $m = 1$