Binomial Functions

Binomial Series

$$\binom{n}{r}+\binom{n}{r+1}=\binom{n+1}{r+1}$$

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^n\mathrm{C}_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1.2\ldots r}x^r + \ldots \qquad (|x|<1, n \in \mathbb{R})$$

Q1

Find the term in x^3 in the binomial expansion of

$$(1 + 2x)^{-1}$$ [4]

Q2

Find the first 3 terms in the binomial expansion of

$$\frac{1}{(3-x)^2}$$ [7]

Q3

Find the binomial expansion of

$$\sqrt{1-x^2}$$

up to and including the term in x^4 [6]

Q4

Use the binomial theorem to expand in ascending powers of x

$$\frac{1+x}{(1-x)^{\frac{1}{2}}}$$

up to and including the term in x^2 [7]

Q5

(i) Express

$$\frac{x-2}{(x+2)(x+1)}$$

in partial fractions. [6]

(ii) Hence expand

$$\frac{x-2}{(x+2)(x+1)}$$

in ascending powers of x, as far as the term in x^3, using the binomial theorem. [11]

(iii) Find the range of values of x for which the expansion is valid. [3]

Q6

Use the binomial theorem to expand

$$(1 + x + x^2)^{-1}$$

in ascending powers of x up to and including the term in x^3 [6]