Differential Equations

Q1

Given the differential equation

$$\frac{dy}{dx} = \frac{3y}{x+1}$$

and that $x = 1$ when $y = 16$, express y in terms of x [10]

Q2

Solve the differential equation

$$\frac{1}{2}\frac{dy}{dx} = \frac{x\sqrt{y}}{x-1}$$

given that $y = 4$ when $x = 2$ [9]

Q3

Solve the differential equation

$$\left(\sin^2 \theta\right)\frac{\mathrm{d}x}{\mathrm{d}\theta} = \frac{4}{x^2}$$

given that $x = 3$ when $\theta = \frac{\pi}{4}$ [7]

Q4

The voltage V of the battery in a smoke alarm can be modelled by

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -kV$$

where t is the number of days after installation and k is a positive constant.
The initial voltage is 9 volts and after 30 days the voltage drops to 4.5 volts.

(i) Solve this differential equation. [7]

The smoke alarm does not function properly if the battery's voltage drops below 1.5 volts.

(ii) Find after how many days the battery should be changed. [2]

Q5

In the atmosphere, the air pressure P (Pascals) decreases with the height h (km) above sea level at a rate that is proportional to the pressure.

(i) Model this by a differential equation. [2]

At sea level the air pressure is 100 000 Pa.
At 1 km above sea level the air pressure is 88 000 Pa.

(ii) By solving the differential equation, find the air pressure at 400 m above sea level. [8]

Q6

While on a skiing holiday in the Alps, Ben pours a cup of hot coffee from his flask.
The initial temperature of the coffee is 85 °C and the temperature of Ben's surroundings is a constant 10 °C.
The rate of change of the temperature x °C of the coffee after time t minutes ($t \geqslant 0$) is proportional to the difference in temperature between the coffee and the surroundings.

(i) Show that $\frac{dx}{dt} = -k(x - 10)$

where k is a positive constant. [3]

After two minutes, Ben's coffee has cooled to 65 °C.

(ii) Find the temperature of Ben's coffee after one **further** minute. [9]

(iii) Find t when $x = 35$ [3]