Lagan College Mathematics Department

GCSE
FURTHER MATHS

Simultaneous Equations

PP Questions

10 Alison had twelve £2 coins, eight £1 coins and twenty 20p coins. The total mass of the coins was 320 g.

Let x, y and z represent the masses, in grams, of a £2 coin, a £1 coin and a 20p coin respectively.

(i) Show that x, y and z satisfy the equation

$$3x + 2y + 5z = 80 \qquad [1]$$

Brian had twenty-five £2 coins, thirty £1 coins and fifteen 20p coins.
The total mass of these coins was 660 g.

(ii) Show that x, y and z also satisfy the equation

$$5x + 6y + 3z = 132 \qquad [1]$$

Christine had eighteen 50p coins, twelve £1 coins and twenty-seven 20p coins.
The total mass of these coins was 393 g.

The mass of a 50p coin is $\frac{2}{3}$ that of a £2 coin.

(iii) Show that x, y and z also satisfy the equation

$$4x + 4y + 9z = 131 \qquad [2]$$

(iv) Solve these equations to find the masses of **all four** coins, i.e. a £2 coin, a £1 coin, a 50p coin and a 20p coin. Show clearly each stage of your solution. [8]

David had twenty £2 coins, some of which were counterfeit. Each counterfeit coin has a mass of 10 g. The total mass of David's coins was 228 g.

(v) Calculate how many counterfeit coins David had. [2]

10 Callum takes either Cornflakes or Porridge or Crispies for breakfast. The number of grams of carbohydrate, protein and fibre in each 100 g portion of these breakfasts is given in **Table 2**.

Table 2

	Cornflakes	**Porridge**	**Crispies**
Carbohydrate	84	60	68
Protein	7	11	6
Fibre	3	9	1

Callum found that in a month his breakfasts gave him a total of 824 g of carbohydrate, 100 g of protein and 58 g of fibre.

Let x, y and z represent the amounts, in hundreds of grams, of Cornflakes, Porridge and Crispies that he ate in the month.

(i) By considering the amounts of carbohydrate in each breakfast show that x, y and z satisfy the equation

$$21x + 15y + 17z = 206 \qquad [2]$$

(ii) By considering the amounts of protein and fibre in each breakfast write down two more equations satisfied by x, y and z. [2]

(iii) Solve these equations, showing clearly each stage of your solution, to find the **amounts, in grams**, of Cornflakes, Porridge and Crispies that Callum ate in the month. [8]

David takes either Weetabicks or Shredded Weet for his breakfast. The number of grams of protein and fibre in each 100 g portion of these breakfasts is given in **Table 3**.

Table 3

	Weetabicks	**Shredded Weet**
Protein	12	9
Fibre	10	7.5

David found that during the month his breakfasts gave him a total of 120 g of protein and 100 g of fibre.

Let p and q represent the amounts, in hundreds of grams, of Weetabicks and Shredded Weet that David ate in the month.

(iv) Write down two equations satisfied by p and q and explain why it is not possible to determine from these equations the amounts of Weetabicks and Shredded Weet that David ate. [2]

10 Matthew, Emma and Simon decided to invest some money in low, medium and high risk accounts.

Matthew invested £5 000, £3 000 and £2 000 in the low, medium and high risk accounts respectively, and his expected interest after one year is £500.

Let x, y and z represent the **percentage** interest rates for the low, medium and high risk accounts respectively.

(i) Show that x, y and z satisfy the equation

$$5x + 3y + 2z = 50 \qquad [1]$$

Emma invested £2 000, £2 000 and £7 000 in the low, medium and high risk accounts respectively. Her expected interest after one year is £860.

(ii) Show that x, y and z also satisfy the equation

$$2x + 2y + 7z = 86 \qquad [1]$$

Simon invested £8 000, £2 000 and £10 000 in the low, medium and high risk accounts respectively. His expected interest after one year is £1 340.

(iii) Show that x, y and z also satisfy the equation

$$4x + y + 5z = 67 \qquad [1]$$

(iv) Solve these equations, showing clearly each stage of your solution, to find the **percentage** interest rates for the low, medium and high risk accounts. [7]

Penny wants to invest a total of £10 000 in the medium and high risk accounts such that she can expect to earn a total of £800 in interest after one year.

Let a and b represent the amounts she should invest in the medium and high risk accounts respectively.

(v) Write down two equations satisfied by a and b. [3]

(vi) Solve these equations to determine how much she should invest in each account. [1]

10 Joan regularly takes the train to visit friends in Oldtown, Newtown and Hightown.

In the first quarter of the year she made 10 journeys to Oldtown, 8 to Newtown and 6 to Hightown. The total cost of her train fares was £178

Let x, y and z represent the train fares in pounds to Oldtown, Newtown and Hightown respectively.

(i) Show that x, y and z satisfy the equation

$$5x + 4y + 3z = 89$$ [1]

In the second quarter she made 9 journeys to Oldtown, 9 to Newtown and 15 to Hightown. The total cost of her fares was £219

(ii) Show that x, y and z also satisfy the equation

$$3x + 3y + 5z = 73$$ [1]

In the third quarter of the year she made 7 journeys to Oldtown, 5 to Newtown and 6 to Hightown. The total cost of her fares was £130

(iii) Write down a third equation satisfied by x, y and z. [1]

(iv) Solve these equations, showing clearly each stage of your solution, to find the train fares to each of the towns. [8]

In the last quarter of the year Joan made an equal number of journeys to Oldtown and Newtown. She made one less journey to Hightown than she did to each of the other towns. The total cost of her fares was £163

Let n represent the number of journeys she made to Oldtown.

(v) Write down an equation satisfied by n. [1]

(vi) Solve this equation and hence determine the number of journeys she made to **each** of the three towns in the last quarter of the year. [2]

10 A wholesaler provides bags of coal, bags of logs and bags of peat briquettes, priced at £x, £y and £z per bag respectively, to three local garages.

Garage A purchases 150 bags of coal, 100 bags of logs and 50 bags of peat briquettes at a total cost of £2250

(i) Show that x, y and z satisfy the equation

$$3x + 2y + z = 45 \qquad [1]$$

Garage B purchases 195 bags of coal, 170 bags of logs and 75 bags of peat briquettes at a total cost of £3195

(ii) Show that x, y and z also satisfy the equation

$$39x + 34y + 15z = 639 \qquad [1]$$

Garage C purchases 150 bags of coal, 75 bags of logs and 60 bags of peat briquettes at a total cost of £2130

(iii) Show that x, y and z also satisfy the equation

$$10x + 5y + 4z = 142 \qquad [1]$$

(iv) Solve these equations, showing clearly each stage of your solution, to find the cost of a bag of coal, a bag of logs and a bag of peat briquettes. [8]

As a special offer, the wholesaler is offering 25% off the price of a bag of coal and 25% off the price of a bag of logs.

Garage C puts in a **new** order to avail of the special offer. This order includes 75 bags of logs and 60 bags of peat briquettes. The wholesaler is asked to complete the order with bags of coal, so that the total cost is again £2130

(v) Calculate how many bags of coal are in the new order for garage C. [3]