Geometric Progressions

Q1

Geometric Series

$u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

The first four terms of a geometric series are

$$0.51,\ 0.0051,\ 0.000051,\ 0.00000051,\ \ldots$$

(i) Find the common ratio. [2]

(ii) Hence by finding the sum to infinity of this series, express the recurring decimal

$$0.5151515151\ldots$$

as a fraction in its simplest form. [4]

Q2

A country's population at the end of each year is 5% greater than at the start of that year.
Model the population to be increasing at a constant rate.

(i) Find an expression for the population after t years. [2]

(ii) Find how many years it will take for the population to increase by 50% [5]

Q3

The time intervals $t_1, t_2, t_3, \ldots$ between consecutive bounces of a ball, bouncing on the ground, form the terms of a geometric progression.

The time interval t_5 is an eighth of the time interval t_2

(i) Find the common ratio. [4]

The sum of the first 6 time intervals is 6.3 seconds.

(ii) Find t_1 [3]

Q4

(i) A sequence is defined recursively by

$$u_{n+1} = \frac{2}{3}u_n \quad \text{where } u_1 = 1$$

Find u_2, u_3 and u_4 [3]

(ii) State whether this sequence is convergent or divergent. [1]

A geometric series is formed by adding the terms of the sequence to give

$$1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + ...$$

(iii) Find the common ratio of this geometric series. [1]

(iv) Find the sum to infinity of this geometric series. [2]

Q5

A sequence is defined by

$$u_{n+1} = 2bu_n \quad u_1 = 6 \quad (n = 1, 2, 3, 4 \ldots)$$

(i) Find u_2 and u_3 in terms of b [3]

(ii) Find the range of values of b for which the sequence converges. [3]