Implicit Differentiation

Q1

A curve has the equation

$$4y - x = xy$$

(i) Show that $\frac{dy}{dx} = \frac{y + 1}{4 - x}$ [5]

(ii) Find the equation of the tangent to the curve at the point (3, 3). [3]

Q2

A curve is defined by

$$3x^2 + xy - 2y^2 = 0$$

(i) Show that

$$\frac{dy}{dx} = \frac{y + 6x}{4y - x}$$ [6]

(ii) Find the equation of the normal to the curve at the point (2, 3). [4]

Q3

Find the equation of the normal to the curve

$$x^2 - 4xy + y^2 = 13$$

at the point $(2, -1)$. [9]

Q4

Part of the graph of

$$2\sin x - x = \tan y$$

is shown in **Fig. 1** below.

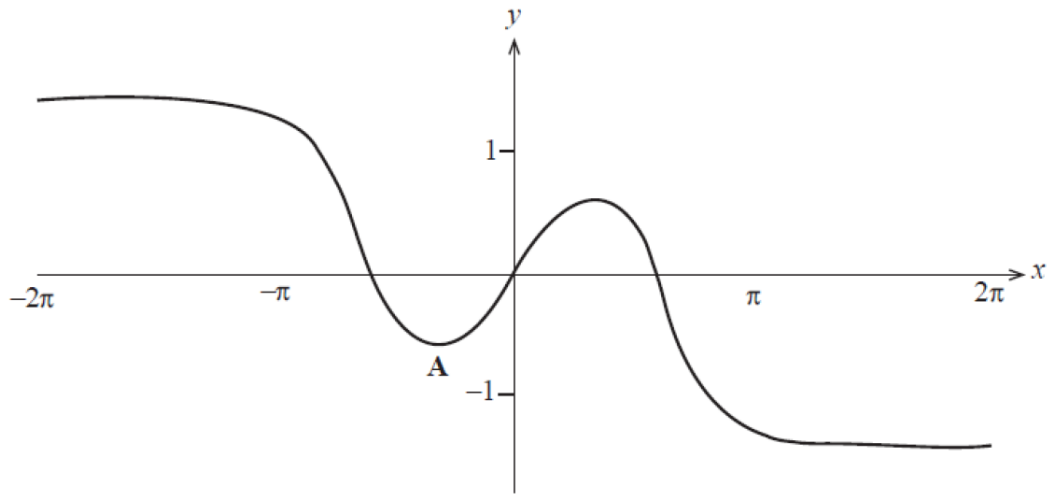

Fig. 1

(i) Find $\frac{dy}{dx}$ [4]

(ii) Hence find the coordinates of the turning point labelled **A** in **Fig. 1** above. [5]

(iii) State the equations of the 2 horizontal asymptotes. [2]

Q5

A curve has the equation

$$y\mathrm{e}^{-2x} = 2x + y^2$$

(i) Show that the gradient function of this curve is given by

$$\frac{2 + 2y\mathrm{e}^{-2x}}{\mathrm{e}^{-2x} - 2y}$$ [7]

The point P (0,1) lies on this curve.

(ii) Find the equation of the normal to this curve at the point P.
Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [4]

Q6

(i) Differentiate

$$x^3 - 3x^2y + 2y^2 = 3$$

implicitly with respect to x. [5]

(ii) Hence find the equation of the tangent to the curve

$$x^3 - 3x^2y + 2y^2 = 3$$

at the point (1, 2). [3]