Integration part 2

Q1

Using the substitution $x = \sqrt{2}\sin\theta$, or otherwise, show that

$$\int_0^1 \sqrt{2 - x^2}\, dx = \frac{2 + \pi}{4}$$ [12]

Q2

The designers of a bowl use an area rotated through 2π radians around the x-axis as the basis for their design.
The area used is between the curve $y = 4\sqrt{x + 2}$, the x-axis and the lines $x = 0$ and $x = a$, as shown in **Fig. 1** below.

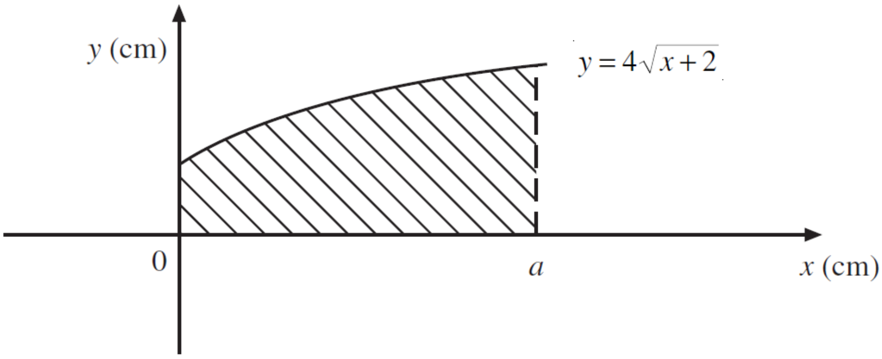

Fig. 1

(i) Find an expression for the capacity of the bowl in terms of a. [7]

The specification requires the capacity of the bowl to be $1000\,\text{cm}^3$

(ii) Find the value of a correct to one decimal place. [7]

Q3

(a) Find

$$\int 16x^3 \ln x \, \mathrm{d}x$$ [7]

(b) Find

$$\int (\sin x + \cos x)^2 \, \mathrm{d}x$$ [6]

Q4

(a) Using the substitution $u = 2 + x$ find

$$\int x(2 + x)^{10}\,\mathrm{d}x$$ [7]

(b) Find the **exact** value of

$$\int_0^{\frac{\pi}{4}} 8x\cos 2x\,\mathrm{d}x$$ [6]

Q5

(a) Find $\int \sin^3 x \, dx$ [5]

(b) Find the **exact** value of the volume generated when the area bounded by the curve $y = 2e^x$, the x-axis and the lines $x = 1$ and $x = 4$ is rotated through 2π radians about the x-axis. [7]

Q6

A bowl is formed by rotating through 2π radians about the x-axis, the arc of the curve

$$y = \sqrt{5x}$$

between $x = 0$ and $x = a$, where a is a positive constant.
The bowl is full of water.
Find the volume of water in the bowl. [6]

Q7

Using the substitution $u = 1 + x$, find the **exact** value of

$$\int_{-1}^{0} x(1+x)^{\frac{1}{2}}\,\mathrm{d}x$$

[8]