Newton Raphson

Q1

Numerical Solution of Equations

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

(i) Show that the equation

$$x^3 + 2x - 1 = 0$$

has a root between $x = 0$ and $x = 1$ [3]

(ii) Take the first approximation to this root to be 0.6 and use the Newton–Raphson method twice to find a better approximation to this root. [5]

Q2

The equation $4e^{-x} - x = 0$ has a root which is approximately 1.3
Starting with this value for x, use the Newton-Raphson method twice to find a better approximation to the root. [7]

Q3

(i) Show that the equation $2 - \ln x = x^2$ has a solution between $x = 1$ and $x = 2$ [4]

(ii) By taking $x = 1$ as a first approximation and using the Newton–Raphson method twice, find a better approximation to the solution of the equation $2 - \ln x = x^2$ [5]

Q4

(i) On one diagram sketch the graphs of

$$y = e^x$$

and

$$y = x^2$$ [3]

The x coordinate of the graphs' only point of intersection can be found by solving the equation

$$e^x - x^2 = 0$$

(ii) Verify that this value of x lies between $x = -1$ and $x = 0$ [3]

(iii) Taking $x = -0.6$ as a first approximation to this value of x, use the Newton-Raphson method twice to find a better approximation. [5]

Q5

(i) On one diagram sketch the graphs of

$$y = \ln x$$

and

$$y = 4 - x^2$$

[3]

From **(i)** it can be seen that a root of the equation

$$\ln x + x^2 - 4 = 0$$

lies between $x = 1$ and $x = 2$

(ii) By taking $x = 2$ as a first approximation to this root and using the Newton-Raphson method twice, find a better approximation to this root. [5]