Newton Raphson

Q1

Numerical Solution of Equations

The Newton-Raphson iteration for solving $f(x)=0$: $x_{n+1}=x_n-\dfrac{f(x_n)}{f'(x_n)}$

(i) Show that the equation

$$x^3+2x-1=0$$

has a root between $x=0$ and $x=1$ [3]

(ii) Take the first approximation to this root to be 0.6 and use the Newton–Raphson method twice to find a better approximation to this root. [5]

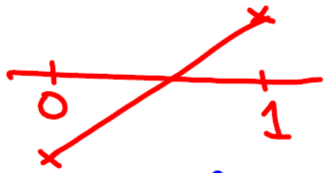
(i) let $f(x)=x^3+2x-1$

Show root in the interval by putting in $x=0$ then $x=1$

$f(0)=0^3+2\times0-1$ $\quad$ $f(1)=1^3+2\times1-1$

$f(0)=-1$ $\quad$ $f(1)=2$

The curve is continuous and there is a CHANGE of sign



(ii) $f(x)=x^3+2x-1$

$f'(x)=3x^2+2$

1st approx $x_1=0.6$

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}=0.6-\frac{0.416}{3.08}=0.4649$$

$$x_3=x_2-\frac{f(x_2)}{f'(x_2)}=0.4649-\frac{0.03037}{2.6484}=0.45343$$

Ans Better approximation $=0.453$ (3 sig. fig.)

Q2

The equation $4e^{-x} - x = 0$ has a root which is approximately 1.3
Starting with this value for x, use the Newton-Raphson method twice to find a better approximation to the root. [7]

$$f(x) = 4e^{-x} - x$$

$$f'(x) = -4e^{-x} - 1$$

Newton Raphson

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 1.3$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.3 - \frac{f(1.3)}{f'(1.3)}$$

$$= 1.3 - \frac{-0.20987}{-2.09013}$$

$$= 1.19959$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.19959 - \frac{f(1.19959)}{f'(1.19959)}$$

$$= 1.19959 - \frac{0.00568}{-2.20527}$$

$$= 1.20217$$

Answer Better Approximation 1.20 3sig.fig.

Q3

(i) Show that the equation $2 - \ln x = x^2$ has a solution between $x = 1$ and $x = 2$ [4]

(ii) By taking $x = 1$ as a first approximation and using the Newton–Raphson method twice, find a better approximation to the solution of the equation $2 - \ln x = x^2$ [5]

$2 - \ln x = x^2$

$0 = x^2 - \ln x - 2$

let $f(x) = x^2 - \ln x - 2$

Look at the values $x = 1$ & $x = 2$

$f(1) = 1^2 - \ln 1 - 2$

$f(1) = -1$

$f(2) = 2^2 + \ln 2 - 2$

$f(2) = 2.693$

Curve is continuous in this interval

There is a change of sign

Therefore there is a root somewhere in the interval $1 < \text{root} < 2$

(ii) Newton-Raphson Method

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

This is on the formula booklet

$f(x) = x^2 + \ln x - 2$

$f'(x) = 2x + \frac{1}{x}$

$x_1 = 1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{-1}{3} = 1.33333$$

$$x_3 = 1.33333 - \frac{f(1.33333)}{f'(1.33333)}$$

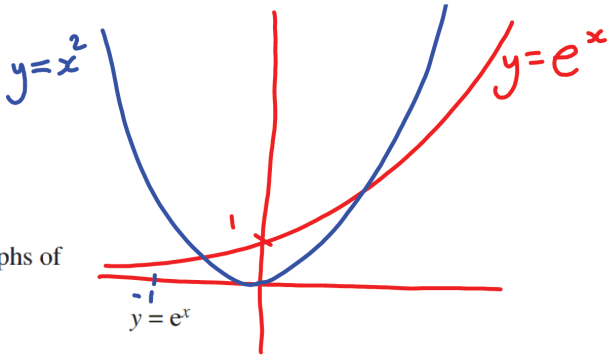
$$x_3 = 1.33333 - \frac{0.06545}{3.41666}$$

$x_3 = 1.31417$

Ans Better Approximation 1.31 3sig.fig

Q4

(i) On one diagram sketch the graphs of



$y = e^x$

and

$y = x^2$ [3]

The x coordinate of the graphs' only point of intersection can be found by solving the equation

$$e^x - x^2 = 0$$

(ii) Verify that this value of x lies between $x = -1$ and $x = 0$ [3]

(iii) Taking $x = -0.6$ as a first approximation to this value of x, use the Newton-Raphson method twice to find a better approximation. [5]

let $f(x) = e^x - x^2$

$f(-1) = e^{-1} - (-1)^2 = -0.6321$

$f(0) = e^0 - (0)^2 = 2.7183$

Change of Sign and Continuous Curve

so then there MUST be a root or solution in the interval $-1 < \text{root} < 0$

(iii) If $x_1 = -0.6$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\boxed{\begin{aligned} f(x) &= e^x - x^2 \\ f'(x) &= e^x - 2x \end{aligned}}$$

$$x_2 = -0.6 - \frac{0.18881}{1.74881}$$

$$x_2 = -0.70796$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = -0.70796 - \frac{f(-0.70796)}{f'(-0.70796)}$$

$$x_3 = -0.70796 - \frac{-0.00856}{1.90857}$$

$$x_3 = -0.70347$$

Answer -0.703 (3sig.fig.)

Q5

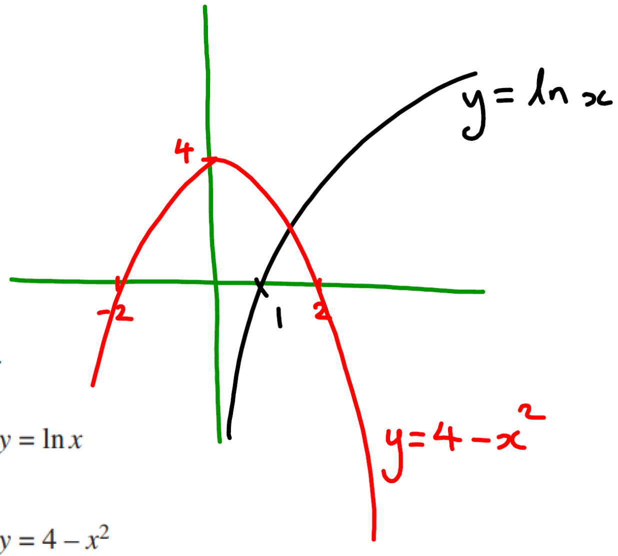
(i) On one diagram sketch the graphs of

$$y = \ln x$$

and

$$y = 4 - x^2$$

[3]



From **(i)** it can be seen that a root of the equation

$$\ln x + x^2 - 4 = 0$$

lies between $x = 1$ and $x = 2$

(ii) By taking $x = 2$ as a first approximation to this root and using the Newton-Raphson method twice, find a better approximation to this root. [5]

let $f(x) = \ln(x) + x^2 - 4$

$$f'(x) = \frac{1}{x} + 2x$$

$x_1 = 2$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 2 - \frac{f(2)}{f'(2)}$$

$$x_2 = 2 - \frac{0.69315}{4.5}$$

$$x_2 = 1.84597$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.84597 - \frac{f(1.84597)}{f'(1.84597)}$$

$$x_3 = 1.84597 - \frac{0.02061}{4.23366}$$

$$x_3 = 1.84110$$

Answer: Better Approx. = 1.84 (3sig.fig.)