Parametric differentiation

Q1

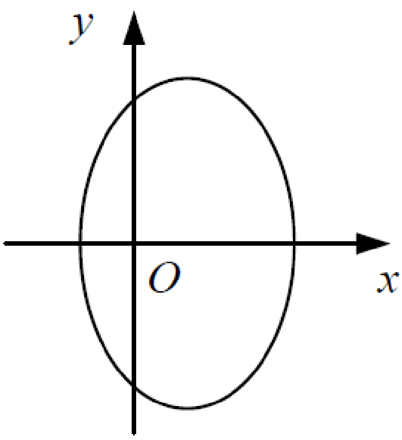
A curve has parametric equations

$$x = \sin^2 t, \quad y = \tan t, \quad -\tfrac{\pi}{2} < t < \tfrac{\pi}{2}.$$

a Show that the tangent to the curve at the point where $t = \frac{\pi}{4}$ passes through the origin.

b Find a cartesian equation for the curve in the form $y^2 = \mathrm{f}(x)$.

Q2



The diagram shows the ellipse with parametric equations

$$x = 1 - 2\cos\theta,\quad y = 3\sin\theta,\quad 0 \le \theta < 2\pi.$$

a Find $\frac{dy}{dx}$ in terms of θ.

b Find the coordinates of the points where the tangent to the curve is

i parallel to the x-axis,

ii parallel to the y-axis.

Q3

A curve has the parametric equations

$$x = \tan^2 t, \qquad y = \cos t, \qquad 0 < t < \frac{\pi}{2}$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t. **(3)**

(b) Find an equation of the tangent to the curve when $t = \frac{\pi}{4}$ **(5)**

(c) Find a cartesian equation for the curve. **(4)**

Q4

A curve is defined by the parametric equations

$$x = 2t \qquad\qquad y = t^3 - 3t$$

Find in terms of t

(i) $\dfrac{dy}{dx}$ [4]

(ii) $\dfrac{d^2y}{dx^2}$ [3]

Q5

A curve is given by the parametric equations

$$x = \sin\theta, \quad y = \sin 2\theta, \quad 0 \leq \theta \leq \tfrac{\pi}{2}.$$

a Find the coordinates of any points where the curve meets the coordinate axes.

b Find an equation for the tangent to the curve that is parallel to the x-axis.

c Find a cartesian equation for the curve in the form $y = \mathrm{f}(x)$.

Q6

A curve is defined parametrically by

$$x = 2\sin\theta + 1 \qquad y = 1 - \cos 2\theta$$

(i) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2\sin\theta$$ [5]

(ii) Find the equation of the tangent to the curve at the point where $\theta = \frac{\pi}{4}$ [5]

(iii) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ [3]