

Partial Fractions

Q1

Express

$$\frac{3x+3}{(x-1)(x+2)}$$

proper fraction $\frac{x+\dots}{x^2+\dots}$

in partial fractions.

[6]

$$\frac{3x+3}{(x-1)(x+2)} \equiv \frac{A}{(x-1)} + \frac{B}{(x+2)}$$

$$\frac{3x+3}{(\quad)(\quad)} \equiv \frac{A(x+2)}{(\quad)(\quad)} + \frac{B(x-1)}{(\quad)(\quad)}$$

$$3x+3 = A(x+2) + B(x-1)$$

Put in $x=1$ $6 = 3A \longrightarrow A=2$

Equate x terms $3 = A + B$

$$3 = 2 + B \longrightarrow B=1$$

Ans $\frac{2}{(x-1)} + \frac{1}{x+2}$

Q2

Write in partial fractions

$$\frac{x}{x^2 - 5x + 6}$$

proper

$$\frac{x + \dots}{x^2 + \dots}$$

[6]

But we must factorise denominator first

$$x^2 - 5x + 6 = (x - 3)(x - 2)$$

Now,
$$\frac{x}{x^2 - 5x + 6} \equiv \frac{x}{(x - 3)(x - 2)} \equiv \frac{A}{(x - 3)} + \frac{B}{(x - 2)}$$

$$\frac{x}{(x)} \equiv \frac{A(x - 2)}{(x)} + \frac{B(x - 3)}{(x)}$$

Remove all denominators

$$x = A(x - 2) + B(x - 3)$$

Now put in $x = 2$ $2 = A(0) + B(-1) \longrightarrow B = -2$

Now put in $x = 3$ $3 = A(1) + B(0) \longrightarrow A = 3$

Answer
$$\frac{3}{(x - 3)} - \frac{2}{(x - 2)}$$

Q3

Express as partial fractions

$$\frac{5x+2}{x^2-4}$$

[6]

Express as partial fractions

$$\frac{5x+2}{x^2-4}$$

proper fraction

$$\frac{5x+\dots}{x^2+\dots}$$

But x^2-4
D.O.T.S.

$$\frac{5x+2}{(x+2)(x-2)} \equiv \frac{A}{(x+2)} + \frac{B}{(x-2)}$$

get a common denominator

$$\frac{5x+2}{(x)(x)} \equiv \frac{A(x-2)}{(x)(x)} + \frac{B(x+2)}{(x)(x)}$$

$$5x+2 = A(x-2) + B(x+2)$$

Put in $x=2$ $12 = A(0) + 4B \longrightarrow B=3$

Equate x terms $5 = A + B \longrightarrow A=2$

Answer $\frac{2}{(x+2)} + \frac{3}{(x-2)}$

Q4

Express $\frac{2x^2-9}{x(x-2)}$ in partial fractions.

[9]

Express $\frac{2x^2-9}{x(x-2)}$ in partial fractions.

Improper fraction

Look $\frac{2x^2}{x^2} = 2$

$$\frac{2x^2-9}{x(x-2)} \equiv \frac{2x^2-9}{x(x-2)} \equiv 2 + \frac{A}{x} + \frac{B}{(x-2)}$$

$$\frac{2x^2-9}{x(x-2)} \equiv \frac{2(x)(x-2)}{x(x-2)} + \frac{A(x-2)}{x(x-2)} + \frac{Bx}{x(x-2)}$$

$$2x^2-9 \equiv 2x(x-2) + A(x-2) + Bx$$

Put $x=0$ $-9 = 0 + -2A + 0$ $A = \frac{9}{2}$

Put $x=2$ $-1 = 2B$ $B = -\frac{1}{2}$

Answer $2 + \frac{9}{2x} - \frac{1}{(x-2)}$

Q5

Express

in partial fractions.

You could do long division, but....

$$\frac{2x^2 - 3x - 29}{x^2 - x - 6}$$

Top Heavy improper fraction

$$\frac{2x^2 + \dots}{x^2 + \dots} = 2 \quad [9]$$

$$\frac{2x^2 - 3x - 29}{(x-3)(x+2)} \equiv 2 + \frac{A}{(x-3)} + \frac{B}{(x+2)}$$

$$\frac{2x^2 - 3x - 29}{(\quad)(\quad)} \equiv \frac{2(x-3)(x+2)}{(\quad)(\quad)} + \frac{A(x+2)}{(\quad)(\quad)} + \frac{B(x-3)}{(\quad)(\quad)}$$

$$2x^2 - 3x - 29 \equiv 2(x-3)(x+2) + A(x+2) + B(x-3)$$

Put in $x=3$

$$2 \times 9 - 9 - 29 = 0 + 5A + 0$$

$$-20 = 5A \quad \longrightarrow \quad A = -4$$

Put in $x=-2$

$$8 + 6 - 29 = 0 + 0 + -5B$$

$$-15 = -5B \quad \quad \quad B = 3$$

Answer

$$2 - \frac{4}{(x-3)} + \frac{3}{(x+2)}$$