R cos(@+a)

Q1

$2 \cos x + 4 \sin x$ can be written in the form

$$R \cos (x - \alpha)$$

where α is acute and R is real.

$R\cos(x-\alpha) \equiv R\cos x \cos\alpha + R\sin x \sin\alpha$

(i) Find R and α. [4]

(ii) Hence solve the equation

$$2 \cos x + 4 \sin x = 3$$

where $0° \leqslant x \leqslant 360°$ [5]

FORMULA BOOKLET

Trigonometric Identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

(i) $2\cos x + 4\sin x \equiv R\cos x\cos\alpha + R\sin x\sin\alpha$

$2 = R\cos\alpha \qquad 4 = R\sin\alpha$

$\frac{2}{R} = \cos\alpha \qquad \frac{4}{R} = \sin\alpha$

Pythagoras'

$\tan\alpha = \frac{4}{2}$

$\alpha = 63.4°$

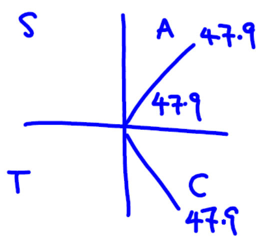
Ans $R = \sqrt{20}$ $\alpha = 63.4°$

(ii) Solve $2\cos x + 4\sin x = 3$

$\sqrt{20}\cos(x - 63.4) = 3$

$\cos(x-63.4) = \frac{3}{\sqrt{20}}$

$\cos m = \frac{3}{\sqrt{20}}$ $\qquad m = 47.9°$ & $312.1°$



$x - 63.4 = 47.9$

$x = 111.3$ ✓

$x - 63.4 = 312.1°$

NOT in interval. $0 <\quad < 360$

What about

$m = -47.9°$

$x - 63.4 = -47.9°$

$x = 15.5°$ ✓

Ans $111.3°$ & $15.5°$

Check $\sqrt{20}\cos(15.5 - 63.4) = 2.99823$

Q2 $r\cos(\theta-\alpha) \equiv r\cos\theta\cos\alpha + r\sin\theta\sin\alpha$

(i) Write $\boxed{3\cos\theta + 4\sin\theta}$ in the form $r\cos(\theta - \alpha)$, where r is real and $0° \leqslant \alpha \leqslant 90°$ [4]

(ii) Hence solve the equation

$$3\cos\theta + 4\sin\theta = -2$$

for $0° \leqslant \theta \leqslant 360°$ [5]

$3\cos\theta + 4\sin\theta \equiv r\cos\theta\cos\alpha + r\sin\theta\sin\alpha$

Trigonometric Identities

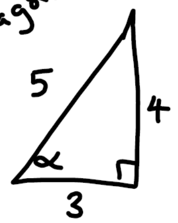
$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$3 = r\cos\alpha$ $\quad$ $4 = r\sin\alpha$

$\frac{3}{r} = \cos\alpha$ $\quad$ $\frac{4}{r} = \sin\alpha$

Pythagoras'



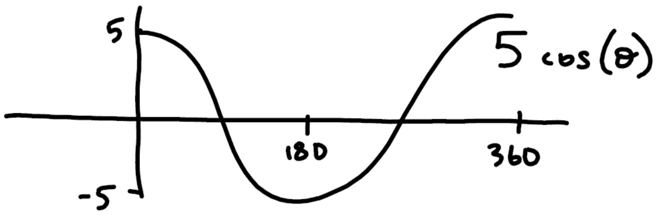
$\tan\alpha = \frac{4}{3}$

$\alpha = \tan^{-1}(\frac{4}{3})$

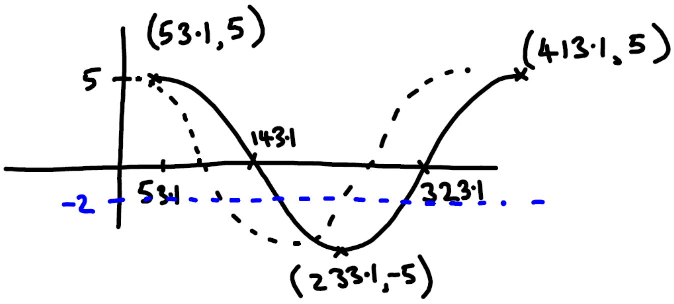
$\alpha = 53.1$

$5\cos(\theta - 53.1°)$

Think about the curve
maxi 5
mini −5



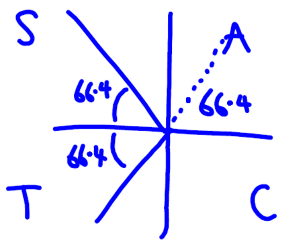
$5\cos(\theta - 53.1°)$
Beyoncé
to the right
by 53.1°



solve $5\cos(\theta - 53.1°) = -2$ Look at - - - - - - line

Should be 2 solutions

$\cos(\theta - 53.1°) = -\frac{2}{5}$



$\cos(m) = -\frac{2}{5}$

$m = 113.6°$ & $246.4°$

$\theta - 53.1 = 113.6$ $\quad$ $\theta - 53.1 = 246.4$

$\theta = 166.7$ Answers $\theta = 299.5$

Q3

(i) Write $4\cos\theta - \sin\theta$ in the form $r\cos(\theta+\alpha)$, where r is real and $0 \leqslant \alpha \leqslant \frac{\pi}{2}$ [4]

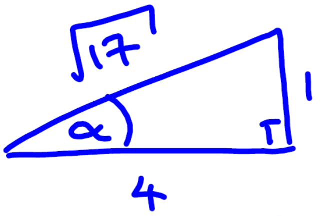
$4\cos\theta - \sin\theta \equiv r\cos\theta\cos\alpha + r\sin\theta\sin\alpha$

(ii) Hence solve the equation

$$4\cos\theta - \sin\theta = 2$$

for $0 \leqslant \theta \leqslant 2\pi$ [5]

$4 = r\cos\alpha$ $\quad 1 = r\sin\alpha$



Ans $r = \sqrt{17}$

$\tan\alpha = \frac{1}{4}$

$\alpha = 0.250$ (3 sig. fig.)

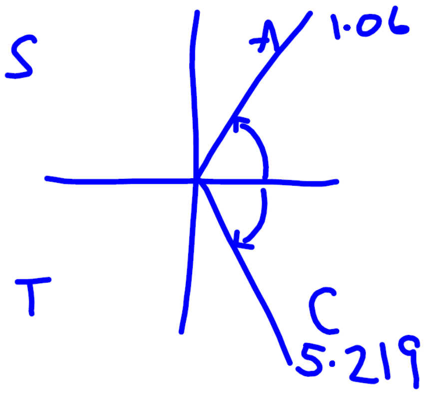
Ans $\sqrt{17}\cos(\theta + 0.250)$

(ii) solve

$\sqrt{17}\cos(\theta+0.250) = 2$

$\cos(\theta+0.250) = \frac{2}{\sqrt{17}}$

solve $\cos(m) = \frac{2}{\sqrt{17}}$



$\theta + 0.250 = 1.06$

$\theta = 0.81$

&

$\theta + 0.250 = 5.219$

$\theta = 4.969$

Answer 0.81 & 4.969

Q4

$\cos x + 2\sin x \equiv r\cos x\cos\alpha + r\sin x\sin\alpha$

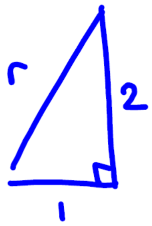
(i) Express $\cos x + 2\sin x$ in the form $r\cos(x - \alpha)$, where $r > 0$ and $0° \leqslant \alpha \leqslant 90°$ [4]

(ii) Hence, or otherwise, find the maximum value of

$$\cos x + 2\sin x$$

and the smallest positive value of x, in degrees, for which it occurs. [3]

(i) $1 = r\cos\alpha$ $\quad 2 = r\sin\alpha$



$r = \sqrt{5}$ Pyth.

$\tan\alpha = \frac{2}{1}$

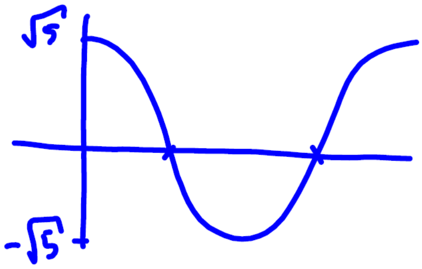
$\alpha = 63.4°$

Ans $\sqrt{5}\cos(x - 63.4°)$

(ii) Maximum value of $\sqrt{5}\cos(x - 63.4°)$

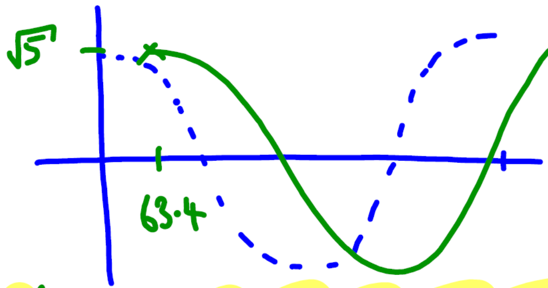
Think cos curve

$\sqrt{5}\cos(x)$



$\sqrt{5}\cos(x - 63.4°)$ Beyoncé

moves "curve to the right"



Maximum is $\sqrt{5}$ at $x = 63.4°$

Check it out

Calculator $\sqrt{5}\cos(63.4 - 63.4)$

Answer $\sqrt{5}$

Q5 Q5

$8\sin\theta + 6\cos\theta \equiv R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

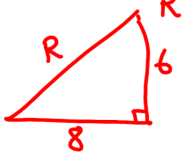
(i) Rewrite $(8\sin\theta + 6\cos\theta)$ in the form

$$R\sin(\theta + \alpha)$$

where R is an integer and $0 \leqslant \alpha \leqslant \frac{\pi}{2}$ [3]

$8 = R\cos\alpha$ $\quad 6 = R\sin\alpha$

$\frac{8}{R} = \cos\alpha$ $\quad \frac{6}{R} = \sin\alpha$



(ii) Hence state the maximum and minimum values of

$$8\sin\theta + 6\cos\theta$$ [2]

$R = 10$ Pythagoras'

$\tan\alpha = \frac{6}{8}$

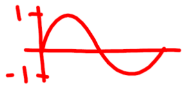
$\alpha = 0.644$ radians

Answer $10\sin(\theta + 0.644)$

(ii) Maximum is $+10$

Minimum is -10

Think about $\sin(\ )$ curve



(iii) A mass is suspended from the end of a spring, as shown in **Fig. 1** below.

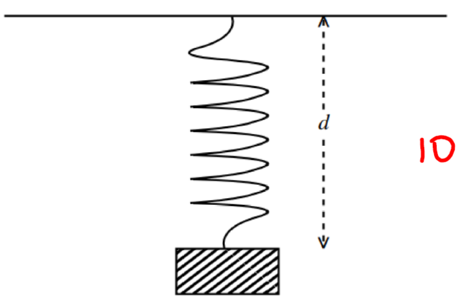


Fig. 1

$10\sin(x + 0.644)$

The mass is oscillating.
After t seconds the distance d (cm) between the fixed point P and the mass is given by

$$d = 15 + 8\sin 2t + 6\cos 2t$$

Find the time at which the mass is first at its lowest point. [4]

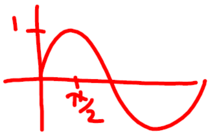
$d = 15 + 8\sin 2t + 6\cos 2t$

$d = 15 + 10\sin(2t + 0.644)$

Lowest point when $d = 15 + 10 = 25$

so when $\sin(2t + 0.644) = 1$

when $\sin\left(\frac{\pi}{2}\right) = 1$



$2t + 0.644 = \frac{\pi}{2}$

$2t = 0.927$

$t = 0.463$ ✓

Ans time 0.463 seconds.

checked $\sin(\ ) = 1$