when there are n trials or experiments

when there are only 2 things can happen $p + q = 1$

Binomial

Q1

Kate sometimes takes sandwiches to school for lunch.

The probability that she takes sandwiches to school for lunch on a particular day is 0.37

(a) Write down the probability that Kate does not take sandwiches to school on a particular day.

$1 - 0.37$

$= 0.63$

Answer 0.63 [1]

Kate wishes to use the Binomial Distribution with $(p + q)^n$ as a model for the number of days she takes sandwiches to school for lunch during a 5-day school week.

(b) For Kate's model, write down the values of n and p.

p = sandwich

q = not sandwich

n = number of days

$n =$ 5 [1]

$p =$ 0.37 [1]

In part **(c)** you may use

$$(p + q)^5 = p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$$

(c) During a 5-day school week, calculate the probability that Kate takes sandwiches to school for lunch:

(i) only once;

sandwich only once

p

$5pq^4$

$5 \times 0.37 \times (0.63)^4$

Answer 0.291 [2]

(ii) at least once.

$p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4$

this is a lot of calculation

but also

$= 1 -$ no sandwiches

$= 1 - q^5 \quad = 1 - (0.63)^5$

Answer 0.901 [3]

Members of a large fitness club can use different methods to pay their membership fees.

Q2 Over time, it is estimated that 35% of members pay their fees using cash.

During one afternoon, six members pay their membership fees.

(a) Explain why the binomial distribution can be used to model the number of these six members who will pay their membership fees using cash.

Binomial can be used because there are only 2 options CASH or NOT CASH. These probabilities add to 1. Also there are 6 trials and prob the same for each of them.

(b) Calculate the probability that exactly three of these six members will pay their membership fees using cash.

You may use

$n = 6$ $\quad p = 0.35$ CASH $\quad q = 0.65$ NOT CASH

$$(p+q)^6 = p^6 + 6p^5q + 15p^4q^2 + 20p^3q^3 + 15p^2q^4 + 6pq^5 + q^6$$

Exactly 3 pay cash

$20p^3q^3$

$20 \times 0.35^3 \times 0.65^3$

0.235

Look at the indices

p^3 means 3 CASH

q^3 means 3 NOT CASH

Answer 0.235 [4]

(c) Calculate the probability that most of these six members will pay their membership fees using cash.

You may use

$$(p+q)^6 = \boxed{p^6 + 6p^5q + 15p^4q^2} + 20p^3q^3 + 15p^2q^4 + 6pq^5 + q^6$$

Most will pay CASH

$p^6 + 6p^5q + 15p^4q^2$

$(0.35)^6 + 6(0.35)^5(0.65) + 15(0.35)^4(0.65)^2$

$0.00184 + 0.0205 + 0.0951$

$= 0.1174$

Ans 0.117

Q3

On a given day, the probability that Michael is on time for work is 0.6

(a) Calculate the probability that Michael is not on time for work on two days in a row.

on time 0.6 $\quad p(\text{on time}) + p(\text{not on time}) = 1$

not on time 0.4

Answer 0.16 [2]

NOT on time AND NOT on time

$0.4 \times 0.4 = 0.16$

(b) **(i)** Write down the name of the most appropriate distribution to model the number of times Michael will be on time for work in a five-day period.

Binomial since fixed number of trials or experiments

2 outcomes

Both outcomes add to 1

Answer BINOMIAL [1]

(ii) For this model, write down the number of trials, n, and the probability of a success, p.

$n =$ 5 [1]

$p =$ on time

$p =$ 0.6 [1]

(c) Calculate the probability that Michael will be on time for work twice in a five-day period.

You may use $(p+q)^5 = p^5 + 5p^4q + 10p^3q^2 + \boxed{10p^2q^3} + 5pq^4 + q^5$

Look for p^2 term

$10 \times p^2 \times q^3$

$10 \times (0.6)^2 \times (0.4)^3$

either

$0.2304 = \frac{144}{625}$

(d) Calculate the probability that Michael will be on time for work at least four times in a five-day period.

You may use $(p+q)^5 = \boxed{p^5 + 5p^4q} + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$

at least 4 on time

look for p^4 term + p^5 term

$5p^4q + p^5$

$5 \times (0.6)^4 \times (0.4) + (0.6)^5 = 0.2592 + 0.1296 = 0.3888$

Q4

$n=6$

Pens are packed in boxes.
There are 6 pens in each box.

$p =$ defective $= 0.1$

The probability that any pen is defective is 0.1.

$q =$ NOT defective $= 0.9$

A box of pens is picked at random.

(*a*) Find the probability that the box contains exactly one defective pen.
Give your answer correct to 3 significant figures.

Clue it is BINOMIAL

You may use $(p + q)^6 = p^6 + 6p^5q + 15p^4q^2 + 20p^3q^3 + 15p^2q^4 + 6pq^5 + q^6$.

Look for p term in big algebra above

$6pq^5 = 6 \times 0.1 \times 0.9^5 =$ 0.354

(2)

(*b*) Find the probability that the box contains at most one defective pen.
Give your answer correct to 3 significant figures.

At most one defective
Look for q^6 term meaning all 6 NOT defective
and the pq^5 term meaning one defective & 5 NOT defective

$q^6 + 6pq^5$
$= (0.9)^6 + 6(0.1) \times (0.9)^5 =$ 0.8857

(2)

Suki buys 125 boxes of pens.

(*c*) Find an estimate for the number of boxes that contain less than two defective pens.

same as part (b) above

125×0.8857
$= 110.7$

111 boxes.

(2)

Q5

Hiki has a biased dice.

The probability that the dice will land on a 6 is 0.2.

Hiki is going to roll the dice 5 times.

(*a*) Work out the probability that the dice will land on a 6 exactly 3 times.
Give your answer correct to 3 decimal places.

You may use $(p + q)^5 = p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$.

$n = 5$ experiments

p = land on a 6 $= 0.2$

q = NOT land on a 6 $= 0.8$

$p + q = 0.2 + 0.8$

$p + q = 1$

Exactly 3 times means look for p^3q^2 term above

$10p^3q^2$

$10 \times (0.2)^3 \times (0.8)^2 = \frac{32}{625}$

0.0512

(3)

(*b*) Work out the probability that the dice will land on a 6 at least once.
Give your answer correct to 3 decimal places.

At least one p

There are lots of term to add together

$p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4$

But this is

$1 -$ no dice land on 6.

$1 -$ prob(all land NOT 6)

$1 - q^5$

$1 - (0.8)^5$

$1 - \frac{1024}{3125} = \frac{2101}{3125} = 0.67233$

(2)