

Standard Deviation

Q1 The grouped frequency table below shows the duration, to the nearest second, for a group of people to find a hidden word in a word search.

| Duration (nearest second) | x | Frequency f | fx | fx^2 |
|---------------------------|-----|---------------|-------------|--------------|
| $0 < t \leq 10$ | 5 | 3 | 15 | 75 |
| $10 < t \leq 20$ | 15 | 14 | 210 | 3150 |
| $20 < t \leq 30$ | 25 | 22 | 550 | 13750 |
| $30 < t \leq 40$ | 35 | 9 | 315 | 11025 |
| $40 < t \leq 50$ | 45 | 2 | 90 | 4050 |
| | | <u>50</u> | <u>1180</u> | <u>32050</u> |

Jane said that someone took 50 seconds to find the hidden word.

(a) Explain why Jane may not be correct.

Jane can't be certain that someone took 50 seconds.

We know 2 people took between $40 \leq t \leq 50$ so they may both have taken $40 < t < 50$

(b) Calculate an estimate of the mean time taken to find the hidden word.

You may use the blank columns to help with your working.

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{1180}{50}$$

Answer 23.6 seconds [4]

(c) Using the statistical functions on your calculator, or otherwise, calculate an estimate of the standard deviation of the times taken.

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$= \sqrt{\frac{32050}{50} - (23.6)^2}$$

Answer 9.17 seconds [2]

$$= \sqrt{641 - 556.96} = \sqrt{84.04} = 9.17$$

Q2

Members of an all-female choir sing in either the soprano or alto section.

The ages, in years, of the members of the alto section of the choir are as follows:

42 23 34 18 22 55 20 36 27 43

(a) Show that the mean age of these members of the choir is 32 years.

$$\text{Mean} = \frac{42 + 23 + 34 + \dots + 43}{10}$$

$$= \frac{320}{10}$$

$$\text{Answer} = 32 \quad [1]$$

(b) Calculate the standard deviation of the ages of these members of the choir.

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left[\frac{\sum fx}{\sum f}\right]^2}$$

$$\sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}$$

Answer _____ [3]

$$\sum x = 42 + 23 + 34 + 18 + \dots = 320$$

$$\sum x^2 = 42^2 + 23^2 + 34^2 + \dots + 43^2 = 11556$$

$$\text{St. Dev} = \sqrt{\frac{11556}{10} - (32)^2}$$

$$= \sqrt{1155.6 - 32^2}$$

$$= 11.47$$

Ans 11.5

Q3

$$\begin{aligned}\text{Standard deviation} &= \sqrt{\frac{\sum fx^2}{\sum f} - \left[\frac{\sum fx}{\sum f}\right]^2} \\ &= \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}\end{aligned}$$

The marks (x) of 30 students in the Religious Studies test are summarised as follows:

$$\sum x = 2124 \quad \sum x^2 = 151887$$

(d) (i) Calculate the mean of the Religious Studies marks.

$$\text{Mean} = \frac{\sum x}{n} = \frac{2124}{30}$$

Answer 70.8 [2]

(ii) Calculate the standard deviation of the Religious Studies marks.

$$\text{St. Dev} = \sqrt{\frac{151887}{30} - (70.8)^2}$$

$$= \sqrt{5062.9 - 5012.64}$$

$$= \sqrt{50.26} \quad \text{Answer } \underline{7.09} \quad [3]$$

$$= 7.09 \quad 2 \text{ dp.}$$

Sam records the typical operating temperature (in °C) of different types of engines as shown in the table below.

Q4

| Temperature (°C) | Frequency | Mid-points x | fx | fx^2 |
|------------------|-----------|-------------------|-------|----------|
| $30 < x \leq 40$ | 4 | 35 | 140 | 4900 |
| $40 < x \leq 45$ | 7 | 42.5 | 297.5 | 12643.75 |
| $45 < x \leq 50$ | 8 | 47.5 | 380 | 18050 |
| $50 < x \leq 60$ | 6 | 55 | 330 | 18150 |
| $60 < x \leq 80$ | 10 | 70 | 700 | 49000 |

$$\sum f = 35$$

$$\sum fx = 1847.5$$

$$\sum fx^2 = 102743.8$$

- (a) Calculate an estimate of the mean temperature of the engines.

Give your answer correct to 3 significant figures.

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{1847.5}{35} = 52.8$$

3 sig. fig.

Answer 52.8 °C [3]

- (b) Show that an estimate of the standard deviation for the temperatures is 12.2 °C.

$$\begin{aligned} \text{St-Dev} &= \sqrt{\frac{\sum fx^2}{\sum f} - (\bar{x})^2} \\ &= \sqrt{\frac{102743.8}{35} - \left(\frac{1847.5}{35}\right)^2} \\ &= 12.215 \end{aligned}$$

[3]

Each sample consists of 10 packets of dried fruit. The masses, x grams, of the packets in one sample are summarised as follows.

Q5

$$\sum x = 492.9 \quad \sum x^2 = 24\,384.5$$

(i) Calculate the mean and standard deviation of these masses.

$$\text{Mean} = \frac{\sum x}{n} = \frac{492.9}{10} = 49.29$$

$$\begin{aligned} \text{St. Dev} &= \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} \\ &= \sqrt{\frac{24384.5}{10} - \left(\frac{492.9}{10}\right)^2} = \sqrt{8.9459} \end{aligned}$$

Mean 49.29 grams

Standard deviation 2.99 grams [3]

(ii) One packet of dried fruit in the sample has been incorrectly recorded as having a mass of 42.15 g. The correct mass of the packet is 52.15 g.

What effect will using the correct mass of this packet of dried fruit have on the answers to (c)(i)?

The mean will

Increase Decrease Stay the same

The standard deviation will

Increase Decrease Stay the same

because it is now [2]

closer to the mean 49.29

42.15 is further away from 49.29

52.15 is closer to 49.29

St. Deviation is a measure of the spread