Trapezium Rule

Numerical integration

The trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0+y_n)+2(y_1+y_2+\ldots+y_{n-1})\}$, where $h=\frac{b-a}{n}$

Q1

Use the trapezium rule with 5 ordinates to find an approximate value for

$$\int_{-4}^{0}\sqrt{4x+16}\,dx \qquad [6]$$

5 ordinates means 4 strips

x	-4	-3	-2	-1	0
$\sqrt{4x+16}$	0.0000	2.0000	2.8284	3.4641	4.000

$$\text{Area} \approx \frac{h}{2}\{y_0+y_n+2(y_1\cdots+y_{n-1})\}$$

$$\approx \frac{h}{2}\{0.0000+4.000+2(2.0000+2.8284+3.4641)\}$$

$$\approx \frac{1.000}{2}\{4.0000+2(8.2925)\}$$

$$\approx \frac{1}{2}\{20.585\}$$

$$\approx 10.2925$$

Ans 10.3 (3 Sig. Fig.)

Q2

Using the trapezium rule with 6 ordinates, find an approximate value for

$$\int_0^1 \frac{4}{(1+x^2)}\,dx \qquad [6]$$

5 strips.

x	0	0.2	0.4	0.6	0.8	1.0
$\frac{4}{(1+x^2)}$	4.0000	3.8462	3.4483	2.9412	2.4390	2.0000

Area $\approx \frac{h}{2}\{\text{ends} + 2(\text{inbetweens})\}$

$\approx \frac{0.2}{2}\{4.0000 + 2.0000 + 2(3.8462 + 3.4483 + 2.9412 + 2.4390)\}$

$\approx 0.1\{6.0000 + 2(12.6747)\}$

≈ 3.13494

3.13 (3s.f.)

Q3

The graph of $y = \sqrt{2+x^2}$ is shown in **Fig. 1** below.

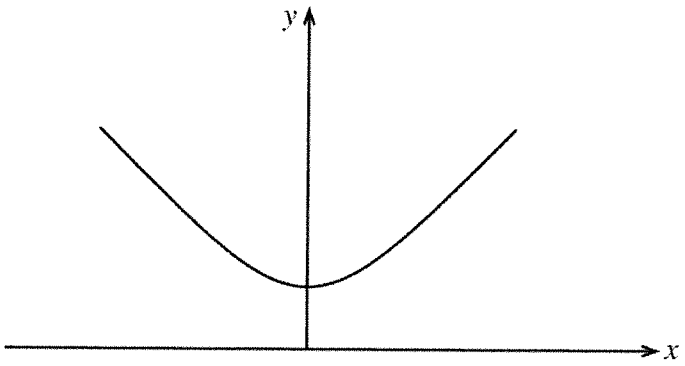

Fig. 1

Use the Trapezium Rule with 6 ordinates to find an approximation for the area bounded by the curve

5 strips

$$y = \sqrt{2+x^2}$$

the x-axis and the lines $x = 0$ and $x = 5$ [6]

$$\int_0^5 \sqrt{2+x^2}\,dx$$

x	0	1	2	3	4	5
$\sqrt{2+x^2}$	1.4142	1.7321	2.4495	3.3166	4.2426	5.1962

Area $\int_0^5 \sqrt{2+x^2}\,dx \approx \frac{h}{2}\{\text{ends} + 2(\text{inbetweens})\}$

$\approx \frac{1}{2}\{1.4142 + 5.1962 + 2(1.7321 + 2.4495 + 3.3166 + 4.2426)\}$

≈ 15.046

Area 151 (3 sig. figs.)

Q4

Use the trapezium rule with 5 ordinates to find an approximate value for

4 strips $\int_0^2 2^x \, dx$ [6]

x	0·0	0·5	1·0	1·5	2·0
2^x	1·000	1·4142	2·0000	2·8284	4·0000

Area $\int_0^2 2^x \, dx \approx \frac{0.5}{2}\{1.0000 + 4.0000 + 2(1.4142 + 2.0000 + 2.8284)\}$

$\approx 0.25\{5.0000 + 2(6.2426)\}$

$\approx 0.25\{17.4852\}$

≈ 4.3713

Ans 4·37 (3 sig. figs)

Q5

4 strips

(i) Use the trapezium rule with five ordinates to find an approximate value for A where

$$A = \int_0^{\frac{\pi}{3}} \tan x \, dx \qquad [6]$$

(ii) Sketch the graph of $y = \tan x$ in the range $0 \leqslant x \leqslant 2\pi$ [2]

(iii) Shade a region on your sketch which has an area approximately equal to A. [1]

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{3\pi}{12}$	$\frac{\pi}{3}$
$\tan x$	0	0.2679	0.5774	1	1.7321

$$\text{Area} = \int_0^{\frac{\pi}{3}} \tan x \, dx \approx \frac{\frac{\pi}{12}}{2}\left\{1.7321 + 2(1.8453)\right\}$$

$$\approx \frac{\pi}{24}\left\{5.4227\right\}$$

$$\approx 0.7098$$

Ans 0.710 (3 sig fig.)

(ii)