# Double and compound angles

## Q1

**Trigonometric Identities**

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\,\pi)$

The acute angles $A$ and $B$ are such that $\tan A = \frac{1}{3}$ and $\cot B = \frac{1}{7}$

Without using a calculator show that $\sin(A - B) = \frac{-2}{\sqrt{5}}$ [6]

# Q2

**(b)** Solve the equation

$$\sin 2\theta = \cos\theta$$

for $-\pi \leqslant \theta \leqslant \pi$ [5]

**(c)** Prove the identity

$$\frac{1 + \tan^2 x}{1 - \tan^2 x} \equiv \sec 2x$$

[7]

# Q3

**(i)** Prove the identity

$$\sin 3A \equiv 3 \sin A - 4 \sin^3 A \qquad [7]$$

**(ii)** Hence solve the equation

$$\sin A + \sin 3A = 0$$

where $0° \leqslant A \leqslant 360°$ [8]

# Q4

**(a)** Prove that

$$\frac{\cos 2A - \cos A + 1}{\sin 2A - \sin A} \equiv \cot A \qquad [5]$$

**(b)** By expressing $\tan 2A$ in terms of $\tan A$, find the **exact** value of $\tan 22\frac{1}{2}^\circ$ [6]

# Q5

The acute angle $A$ is such that $\sin A = \frac{3}{5}$

Using the double angle formulae, find

**(i)** $\sin 2A$ [3]

**(ii)** $\cos 2A$ [2]

# Q6

Prove the identity

$$\frac{1}{\sin 2\theta} + \cot 2\theta \equiv \cot \theta$$