Trigonometric Identities

A+A

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$

Formula

Compound

Double

$\sin 2A = 2\sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 1 - 2\sin^2 A$

$\cos 2A = 2\cos^2 A - 1$

Double and compound angles

Q1

Trigonometric Identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

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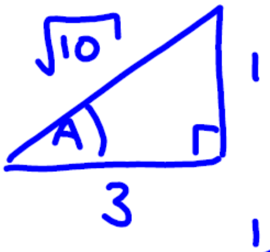
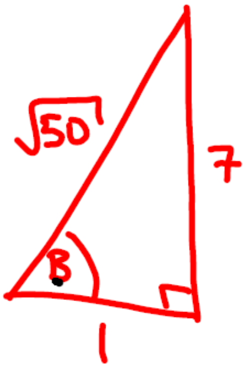
T O A

The acute angles A and B are such that $\tan A = \frac{1}{3}$ and $\cot B = \frac{1}{7}$

Without using a calculator show that $\sin(A - B) = \frac{-2}{\sqrt{5}}$ [6]



$\tan B = \frac{7}{1}$



$\sin A = \frac{1}{\sqrt{10}}$

$\cos A = \frac{3}{\sqrt{10}}$

$\sin B = \frac{7}{\sqrt{50}}$

$\cos B = \frac{1}{\sqrt{50}}$

SOH CAH TOA

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$
$$= \frac{1}{\sqrt{10}}\frac{1}{\sqrt{50}} - \frac{3}{\sqrt{10}}\frac{7}{\sqrt{50}}$$
$$= \frac{1}{\sqrt{500}} - \frac{21}{\sqrt{500}}$$
$$= \frac{-20}{\sqrt{500}} = \frac{-20}{10\sqrt{5}} = \frac{-2}{\sqrt{5}}$$

Q2

(b) Solve the equation $2\sin\theta\cos\theta = \cos\theta$

$\sin 2\theta = \cos\theta$

nasty

for $-\pi \leqslant \theta \leqslant \pi$ [5]

forgot!

(c) Prove the identity

$$\frac{1+\tan^2 x}{1-\tan^2 x} \equiv \sec 2x$$ [7]

$\frac{1}{\cos 2x}$

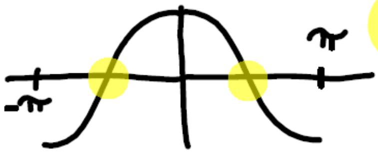
$2\sin\theta\cos\theta = \cos\theta$

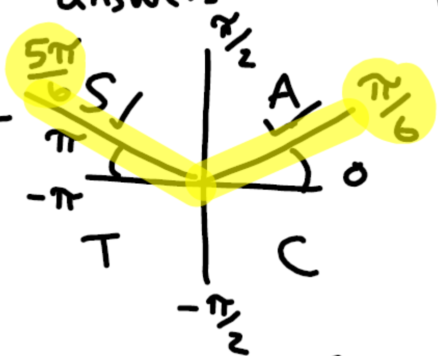
$2\sin\theta\cos\theta - \cos\theta = 0$

$(\cos\theta)(2\sin\theta - 1) = 0$

$\cos\theta = 0$ $\quad\quad$ $\sin\theta = \frac{1}{2}$

answers $\quad\quad$ answers





answer

$\frac{\pi}{2}, -\frac{\pi}{2}$

$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

answers $\frac{\pi}{6}$, $\frac{5\pi}{6}$

$$\frac{1+\tan^2 x}{1-\tan^2 x}$$

$$= \frac{\left(1 + \frac{\sin^2 x}{\cos^2 x}\right)}{1 - \frac{\sin^2 x}{\cos^2 x}}$$

$$= \frac{\left(\frac{\cos^2 x + \sin^2 x}{\cos^2 x}\right)}{\left(\frac{\cos^2 x - \sin^2 x}{\cos^2 x}\right)}$$

$$= \frac{\left(\frac{1}{\cancel{\cos^2 x}}\right)}{\left(\frac{\cos 2x}{\cancel{\cos^2 x}}\right)}$$

$$= \frac{1}{\cos 2x}$$

$1 + \frac{3}{4}$

$\frac{4}{4} + \frac{3}{4}$

$\frac{4+3}{4}$

$\frac{\left(3/7\right)}{\left(5/7\right)}$

$3/5$

Q3

(i) Prove the identity

$$\sin 3A \equiv 3\sin A - 4\sin^3 A \qquad [7]$$

(ii) Hence solve the equation

$$\sin A + \sin 3A = 0$$

where $0° \leqslant A \leqslant 360°$. [8]

$\sin A + 3\sin A - 4\sin^3 A = 0$

$4\sin A - 4\sin^3 A = 0$

(i) $\sin 3A$

$= \sin(2A + A)$

$= \sin 2A \cos A + \cos 2A \sin A$

$= 2\sin A \cos A \cos A + (1 - 2\sin^2 A)\sin A$

$= 2\sin A \cos^2 A + \sin A - 2\sin^3 A$

$= 2\sin A(1 - \sin^2 A) + \sin A - 2\sin^3 A$

$\underline{\underline{2\sin A}} - \underline{2\sin^3 A} + \underline{\underline{\sin A}} - \underline{2\sin^3 A}$

$3\sin A - 4\sin^3 A$ ✓

$4\sin A - 4\sin^3 A = 0$

$(4\sin A)(1 - \sin^2 A) = 0$

$\sin A = 0$

solutions

0, 180, 360

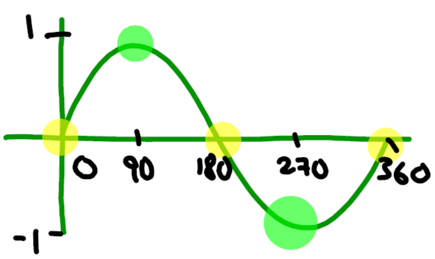
$1 - \sin^2 A = 0$

$\sin^2 A = 1$

$\sin A = \pm 1$

solutions

90 & 270



Q4

(a) Prove that

get rid & the double angles   tricky = easy

$$\frac{\cos 2A - \cos A + 1}{\sin 2A - \sin A} \equiv \cot A \qquad [5]$$

(b) By expressing $\tan 2A$ in terms of $\tan A$, find the **exact** value of $\tan 22\tfrac{1}{2}^\circ$ [6]

$$= \frac{\cos 2A - \cos A + 1}{\sin 2A - \sin A}$$

$$= \frac{2\cos^2 A - 1 - \cos A + 1}{2\sin A\cos A - \sin A}$$

$$= \frac{2\cos^2 A - \cos A}{2\sin A\cos A - \sin A}$$

$$= \frac{\cos A\,(\cancel{2\cos A - 1})}{\sin A\,(\cancel{2\cos A - 1})}$$

$$= \frac{\cos A}{\sin A}$$

$$= \cot A$$

(b)

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\tan 45 = \frac{2\tan(22\tfrac{1}{2})}{1 - \tan^2(22\tfrac{1}{2})}$$

$$1 = \frac{2\tan(22\tfrac{1}{2})}{1 - \tan^2(22\tfrac{1}{2})}$$

$$1 - \tan^2(22\tfrac{1}{2}) = 2\tan(22\tfrac{1}{2})$$

$$0 = \tan^2(22\tfrac{1}{2}) + 2\tan(22\tfrac{1}{2}) - 1$$

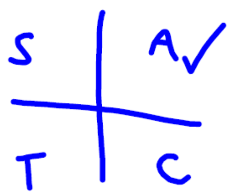
Quadratic like

$$0 = m^2 + 2m - 1$$

solve using quadratic formula

$$m = -1 + \sqrt{2} \quad \text{or} \quad -1 - \sqrt{2}$$

But $\tan(22\tfrac{1}{2})$ is positive



$$\tan(22\tfrac{1}{2}) = -1 + \sqrt{2}$$

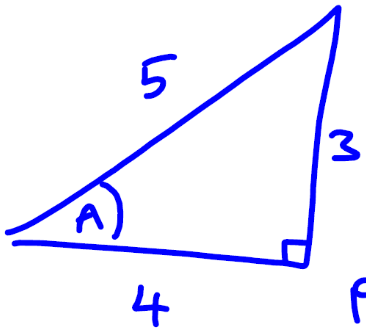
Q5

The acute angle A is such that $\sin A = \frac{3}{5}$

Using the double angle formulae, find

(i) $\sin 2A$ [3]

(ii) $\cos 2A$ [2]



Pythagoras to get 4

$\sin A = \frac{3}{5}$

$\cos A = \frac{4}{5}$

$\sin 2A = 2 \sin A \cos A$

$\sin 2A = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$= \frac{24}{25}$

$\cos 2A = \cos^2 A - \sin^2 A$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25}$

$= \frac{7}{25}$

Q6

Prove the identity

$$\frac{1}{\sin 2\theta} + \cot 2\theta \equiv \cot \theta$$

Get from tricky side to easy side

$$= \frac{1}{\sin 2\theta} + \frac{\cos 2\theta}{\sin 2\theta}$$

$$= \frac{1 + \cos 2\theta}{\sin 2\theta}$$

$$= \frac{1 + 2\cos^2\theta - 1}{2\sin\theta\cos\theta}$$

This helps get rid of the 1

$$= \frac{2\cos^2\theta}{2\sin\theta\cos\theta}$$

$$= \frac{\cancel{2}\cos\theta\cancel{\cos\theta}}{\cancel{2}\sin\theta\cancel{\cos\theta}}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta$$