Volume of Revolution

Q1

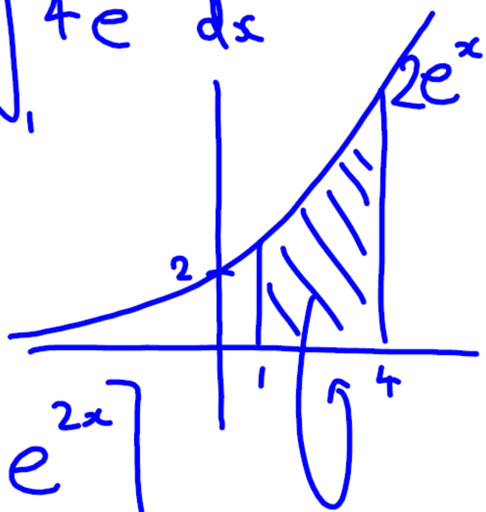
like pottery!

(b) Find the **exact** value of the volume generated when the area bounded by the curve $y = 2e^x$, the x-axis and the lines $x = 1$ and $x = 4$ is rotated through 2π radians about the x-axis. [7]

$$\boxed{\text{Volume} = \int \pi y^2 \, dx}$$

LEARN THIS

$$\text{Volume} = \int_1^4 \pi (2e^x)^2 \, dx = \pi \int_1^4 4e^{2x} \, dx$$



$$\text{Volume} = \pi \left[\frac{4e^{2x}}{2} \right]_1^4$$

$$\text{Volume} = \left[\pi 2e^{2x} \right]^4 - \left[\pi 2e^{2x} \right]_1$$

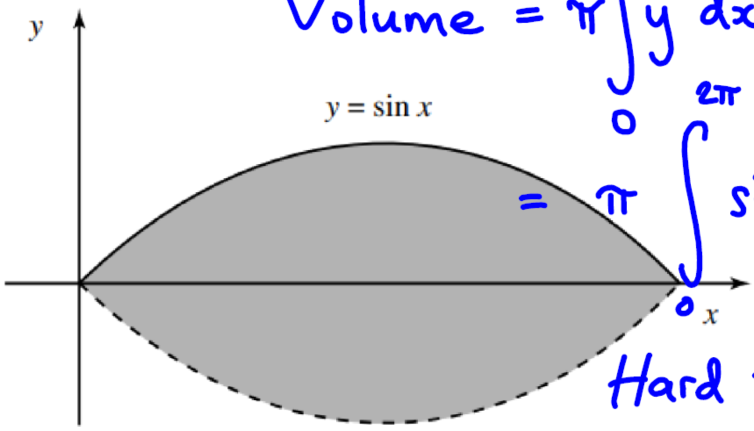
$$\text{Answer} = \underline{\underline{2\pi e^8 - 2\pi e^2}} = 2\pi (e^8 - e^2)$$

Q2

A trophy is to be made in the shape of a rugby ball.
It can be modelled by the volume generated when the area between the curve

$$y = \sin x$$

and the x-axis, between $x = 0$ and $x = \pi$, is rotated through 2π radians about the x-axis, as shown in **Fig. 2** below.



$$\text{Volume} = \pi\int_0^{2\pi} y^2\,dx$$

$$= \pi\int_0^{2\pi} \sin^2 x\,dx$$

Hard to do $\sin^2 x$

Think Double Angles

Fig. 2

Find the **exact** volume of the trophy. [9]

$$\cos 2\theta = 1 - 2\sin^2\theta$$
$$2\sin^2\theta = 1 - \cos 2\theta$$
$$\sin^2\theta = \tfrac{1}{2} - \tfrac{1}{2}\cos 2\theta$$

Now much easier

$$\pi\int_0^{\pi} \frac{1}{2} - \frac{1}{2}\cos 2x\,dx$$

$$= \pi\left[\frac{1}{2}x - \frac{1}{4}\sin 2x\right]^{\pi} - \pi\left[\frac{1}{2}x - \frac{1}{4}\sin 2x\right]_0$$

$$= \pi\left[\frac{\pi}{2} - \frac{1}{4}\sin(2\pi)\right] - \pi\left[0 - 0\right]$$

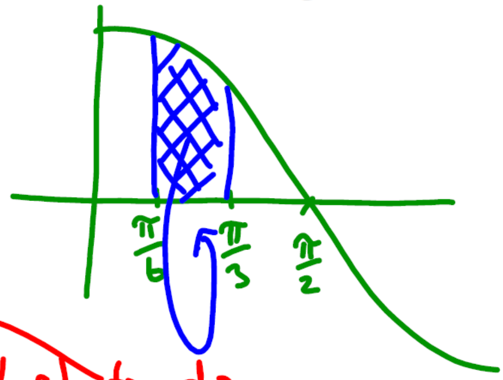
$$= \frac{\pi^2}{2}$$

YOU CAN CHECK USING $\boxed{\int_{\square}^{\square} \square\,dx}$ CALCULATOR BUTTON

Q3

$\int \pi y^2 \, dx$

A paperweight can be modelled by the volume generated when the area between the curve $y = \cos x$, the lines $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ and the x-axis is rotated through 2π radians about the x-axis.



Find the volume of the paperweight. [10]

$$\text{Volume} = \pi \int_{\pi/6}^{\pi/3} \cos^2 x \, dx$$

Hard to do

Use DOUBLE ANGLES

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta + 1 = 2\cos^2\theta$$

$$\frac{1}{2}\cos 2\theta + \frac{1}{2} = \cos^2\theta$$

$$\text{Volume} = \pi \int_{\pi/6}^{\pi/3} \frac{1}{2}\cos 2x + \frac{1}{2} \, dx$$

$$\text{Volume} = \pi \left[\frac{1}{4}\sin 2x + \frac{1}{2}x \right]_{\pi/6}^{\pi/3}$$

$$\text{Volume} = \pi \left[\frac{1}{4}\sin\frac{2\pi}{3} + \frac{1}{2}\left(\frac{\pi}{3}\right) \right] - \pi \left[\frac{1}{4}\sin\frac{2\pi}{6} + \frac{1}{2}\left(\frac{\pi}{6}\right) \right]$$

$$= \pi \left[\frac{\sqrt{3}}{8} + \frac{\pi}{6} \right] - \pi \left[\frac{\sqrt{3}}{8} + \frac{\pi}{12} \right]$$

$$= \pi \left(\frac{\pi}{6} - \frac{\pi}{12} \right)$$

$$= \frac{\pi^2}{12}$$

ASAN!

Check this!! On calculator

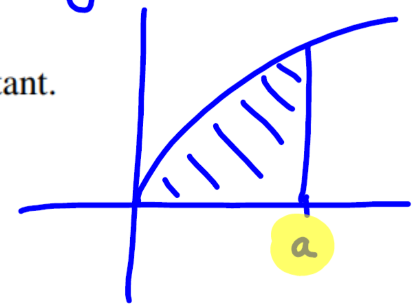
Q4

Volume $= \int \pi y^2 \, dx$

A bowl is formed by rotating through 2π radians about the x-axis, the arc of the curve

$$y = \sqrt{5x}$$

$y^2 = 5x$



between $x = 0$ and $x = a$, where a is a positive constant.
The bowl is full of water.
Find the volume of water in the bowl. [6]

$$\text{Volume} = \pi \int_0^a y^2 \, dx$$

$$\text{Volume} = \pi \int_0^a 5x \, dx$$

$$= \pi \left[\frac{5x^2}{2} \right]_0^a$$

$$= \pi \left[\frac{5a^2}{2} \right] - \left[0 \right]$$

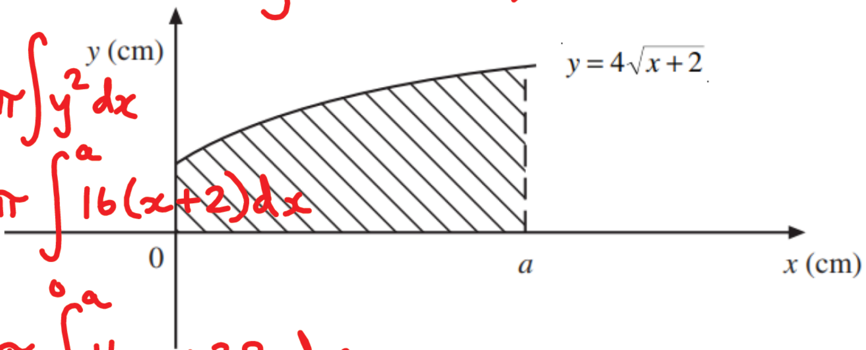
$$= \frac{5a^2\pi}{2}$$

You could check but trickier because you have to pick an a value

Q5

The designers of a bowl use an area rotated through 2π radians around the x-axis as the basis for their design.
The area used is between the curve $y = 4\sqrt{x+2}$, the x-axis and the lines $x = 0$ and $x = a$, as shown in **Fig. 1** below.

$y^2 = 16(x+2)$



$$\text{Volume} = \pi \int y^2\,dx$$

$$= \pi \int_0^a 16(x+2)\,dx$$

$$= \pi \int_0^a 16x + 32\,dx$$

$$= \pi \left[8x^2 + 32x\right]_0^a$$

Fig. 1

(i) Find an expression for the capacity of the bowl in terms of a. [7]

$= \pi(8a^2 + 32a)$ Answer

The specification requires the capacity of the bowl to be 1000 cm^3

(ii) Find the value of a correct to one decimal place. [7]

(ii) Volume = 1000

$$8\pi a^2 + 32\pi a = 1000$$

$$8\pi a^2 + 32\pi a - 1000 = 0$$

Quadratic to find a

Solve either $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ or use calculator buttons

$x = 4.617$ & -8.617

NOT possible

Answer

$x = 4.6$ (1 dec. pl.)

Again check

$$\int_0^{4.16} 16\pi x + 32\pi\,dx$$